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NEBULOSITIES IN MONOCEROS, TAURUS, AND PERSEUS

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ABSTRACT

The present paper is the second of a contemplated series of photographs and studies of the faint diffuse nebulosities, both bright and dark, of the sky. Various problems connected with the photography of these objects, and their reproduction, are outlined. The 3-inch lens adopted for the work appears to have the necessary characteristic of an extended field, with moderate speed. Its efficiency is compared with the Bruce 10-inch doublet. The high-speed plates used have been calibrated with respect to the failure of the reciprocity law, so that the intensities of the various parts of the nebulae can be compared. The effect of preliminary exposure is discussed, and it is shown to be of doubtful utility. A *theory of limiting magnitudes, for stars and nebulae*, is outlined. A *method of reproduction, or intensification*, of the images of the extremely faint extended nebulae sought for has been developed, and it appears to be an improvement over the older methods. Plate VI shows the Milky Way in Monoceros, and is centered on the well-known dark and bright nebulous region surrounding the star 15 Monocerotis. Plates VII and VIII are concerned with the interesting region which includes portions of Taurus and Perseus. A *description of the main features* of the nebulae involved in these constellations is given. The well-known outlying nebulosities of the Pleiades, from a plate taken by Barnard at Mount Wilson, are reproduced separately, and on a scale larger than the preceding.

The present paper, with its accompanying plates, is a continuation of that in the *Astrophysical Journal*, **65**, 137, 1927, which is concerned with the nebulosities in Orion, and is the second of a contemplated series of studies of the extended light and dark nebulosities in the sky. This is a field of stellar photography requiring a lens which can cover an extended angle of view, and at the same time is moderately fast. The Yerkes Observatory is in possession of two such lenses, doublets, of the author's design, both of 3 inches aperture and 21 inches focal length, one corrected for the photographic

and the other for the visual rays, the latter for photovisual photography. They are attached to the mounting of the Bruce 10-inch Petzval and 6-inch Voigtländer lenses. As a rule, exposures are made with all four cameras. Incidentally, it might be mentioned that a program of photographing with these lenses all of the Kapteyn areas from the North Pole to 15° south, with a uniform exposure of four hours, has been initiated at this Observatory, only the best nights being utilized. In addition to the presentation and study of nebulosities, various photographic and photometric problems arising will be discussed.

It is of interest to compare the limiting magnitudes reached by the 10-inch and 3-inch lenses, the exposures being simultaneous. The following results were obtained on the polar sequence, under transparent conditions of winter. The plates, Eastman Speedway, were from the same box.

Exposure Ratio	Time	10-Inch	3-Inch	Diff.
1	89 sec.	$13^{M}.2$	$11^{M}.0$	$2^{M}.2$
9	13.5 min.	15.3	13.7	1.6
81	120 min.	16.4	15.4	1.0

The last column, giving the difference between the limiting magnitudes attained by the lenses, is of especial interest. On the basis of light-grasp, the difference should be 2.6 mag. A very short exposure will probably show this maximum amount. It is noteworthy that the smaller lens gains in efficiency over the larger, when the exposure is prolonged, so that when the exposure time is of two hours' duration, the difference in speed is reduced to only 1 mag. Longer exposures than two hours show no appreciable gain in limiting magnitude with either lens. The reason for the phenomenon is not definitely known. Limiting magnitude is a complicated function of aperture, focal length, exposure time, seeing, sky brightness, and possibly other factors. A partial exposition of the problem is presented below. It is well known that refractors of high-speed ratio, such as our 10-inch doublet, and reflectors, fall off notably in efficiency after the first hour of exposure, nebulosities excepted. In this connection, Isaac Roberts has said:

> The great nebula in Orion has been photographed with the 20-inch reflector at frequent intervals between the years 1886 and 1898 with exposures varying between one minute of time and seven hours thirty-five minutes; yet the stars

are not more numerous or the extensions of the nebulosity greater on the latter than are shown on a plate of like sensitiveness which had been exposed during ninety minutes only; the difference exhibited was that of density[1]

The temptation appeared to be irresistible to Roberts to conclude from this that "the part of the starry universe visible from the earth is limited in extent. . . . ." We now know that the phenomenon is not of cosmical origin, but is photographic, and that a weak light-grasp cannot be overcome by lengthening the exposure, except to a limited extent, the deficiency being greater for stars than for nebulae.

I do not know of attempts to explain in any specific or quantitative way the relation of limiting magnitude to the various factors involved. The following is a suggested explanation of the most important of the phenomena. Let F denote the intensity of the light-flux per unit area on the plate, of the fogging light; S, the same for a star of magnitude m; let a be the aperture, f the focal length, and T the area of the tremor disk. The contrast, C, of the photographic image of the star on its background is, evidently, proportional to the ratio of S plus F to F. Its visibility, V, on the plate is proportional to C, and in addition is a function of the area of the tremor disk. V is also a function of the contrast factor and the graininess of the plate. Under the most favorable conditions, a difference of transparency of only about 1 per cent between image and background is necessary for visibility. But in the case under consideration, of a small object of the size of the tremor disk, the contrast necessary for visibility would be of a much higher order. It will simplify the argument to assume C equal to 2, which is probably not far from the truth, so that S becomes equal to F; that is, the specific intensity of the light in the tremor disk must be equal to the specific intensity of the fogging light, in order that the image may become visible. For a given telescope, this corresponds to a certain stellar magnitude, m_0, which must be the limit which the telescope is capable of rendering, no matter how prolonged the exposure becomes. The only effect of prolonging the exposure is to increase the densities of star and background, without increasing their difference, upon which visibility depends.

[1] *Photographs of Stars, Star Clusters and Nebulae*, **2**, 21, 1899.

It is of interest to apply this principle to a number of cases. Roberts[1] found that the limiting magnitude reached by his 20-inch reflector was about the eighteenth. The limit of the 100-inch mirror at Mount Wilson is approximately the twenty-first magnitude. To make an accurate calculation of the flux F in the two cases requires a knowledge of the relative size of the tremor disks. It is easily seen that even if the 100-inch has a somewhat larger tremor disk, its twenty-five-fold increase of light-grasp, combined with a smaller value of F, due to its favorable location, is quite sufficient to give the observed difference of 3 mag. in limiting magnitudes.

It is instructive to compare a reflector in its Newtonian and Cassegrain forms. If the tremor disk could be kept in the Cassegrain form no larger than in the Newtonian, the former should show a gain of about 2 mag., because of the decrease in the value of F, which is inversely proportional to the square of the focal length, f. The great importance of reducing the size of the tremor disk to its lower possible value is thus seen. For this the essential factors are: degree of optical perfection of the various mirrors, and steadiness and kind of seeing. The various factors which enter into F, the field illumination, aside from the geometrical factor of aperture ratio, are: cleanness of lens and mirror surfaces, absence of flare spots or ghosts, freedom of the glass from bubbles, and other defects. The atmospheric elements are well known and need not be enumerated. There is the further factor of reflection from the glass surface of the photographic plate (p. 289).

It is well known that in photographing nebulosities there is considerable gain in pushing the exposure beyond the two-hour limit which applies to the faint stars. The theory outlined above is capable of explaining this. In the case of extended nebulae, the contrast factor C, at the limit of perception, is considerably less than for stellar images, since the contrast-sensibility of the eye increases rapidly with size of object. The exposure therefore may be prolonged far beyond the limit set for stellar images.

Photographic refractors, of large aperture and small speed-ratio, such as the Thaw refractor of the Allegheny Observatory, and the Yale refractor located in South Africa, form, in this connection, an

[1] *Loc. cit.*

interesting class. The various factors favoring a very low limiting magnitude are strong in this group, namely, the aperture is large, giving a strong light-grasp, the optical correction is very fine, leading to a small tremor disk. Most important of all, the ratio of aperture to focal length is so small that the field brightness F is enormously reduced, entering as a square. With this type of telescope, if the foregoing theory is correct, we should expect that the limiting magnitude would not be reached after two or three hours' exposure, as in the case of reflectors, but would be attained only after an exposure of many times this limit. There is some evidence at the Yerkes Observatory lending support to this prediction. In comparing the relative efficiency of the 40-inch refractor, and the 24-inch reflector stopped to 18 inches, using the same plate and yellow color-screen, Dr. Alice Farnsworth and the writer found from simultaneous exposures that the limiting magnitude was identical for the two instruments for an exposure of one hour's duration, namely, 14.0. For short exposures the reflector is more efficient. Thus, from the curves given by Dr. Farnsworth,[1] we find that in 16 seconds the reflector (18-in. aperture) reaches 8.5, while the refractor attains only 7.7. In other words, as the exposure is prolonged, the refractor gains in efficiency over the reflector. Lending support to this, it has been shown above how the 3-inch lens of aperture ratio $F/7$ gains on the 10-inch lens, of ratio $F/5$, with increasing exposure time.

In photographing the very faint nebulosities, with which the present paper is concerned, it is desirable to use the fastest photographic emulsion obtainable. The Eastman Speedway plate has been adopted, on account of its great speed and uniformity of sensitivity over the surface. For developer, an Eastman formula, D11, has been used. Development time is 10 minutes at a temperature of 70°. It is very desirable to have, for the photometric work which will ultimately be done in forming estimates of the relative brightness of the nebulosities studied, a calibration curve of these plates showing the departure from the so-called "reciprocity law," in connection with the particular developer and development time which has been adopted. In a series of papers in the *Journal of the Optical Society of America* (Vol. **8** ff., 1923), L. A. Jones and associates have made

[1] *Publications of the Yerkes Observatory*, **4**, 218, 1926.

extensive investigations of this subject, which are authoritative and no doubt the last word in a field which heretofore has been full of contradictions, due to faulty procedures and a lack of appreciation of the important part played by the emulsion and developer. Mr. Jones has kindly determined the curve for the Eastman Speedway plate in connection with the developer specified above. Special attention was given by him to the curve applying to very low densities, since this is the important one in the photometry of faint nebulosities. The results are contained in Table I, upon which Figure 1

TABLE I

Log $It+10$

Log $I+10$	$D=0.5$	$D=1.0$	$D=1.5$
4.1	9.33	9.78	10.22
4.7	9.13	9.62	10.09
5.3	8.96	9.48	9.95
5.9	8.79	9.34	9.79
6.5	8.68	9.21	9.65
7.1	8.59	9.10	9.52
7.7	8.53	9.00	9.43
8.3	8.51	8.95	9.38
8.9	8.52	8.95	9.37
9.5	8.55	9.02	9.42
10.1	8.61	9.12	9.51
10.7	8.70	9.23	9.63

is based. In this table, for $D=0.5$, the range of intensity is four millions, corresponding to a range of 16.5 mag. The limits of exposure time are one-hundredth of a second and forty-eight hours. Over this enormous range, the value of gamma or contrast, was found to be constant, 1.17. The values of log It for density of image, $D=0.5$, can be taken as applicable to limiting magnitudes of star images in telescopes, and to faint nebulosities. The results are exceedingly instructive. It can be calculated from Table I that the exposure time corresponding to the optimal intensity, that is, the intensity giving the maximum efficiency, is about 1.6 seconds, and that for a range of about four stellar magnitudes on each side of this point, the reciprocity law holds (Fig. 1). It is important to relate this to exposures with the photographic telescope. It is accomplished by finding the limiting magnitude reached in an exposure of 1.6 sec-

onds. In the case of a few telescopes, rough values are as follows, assuming that the plate is Eastman Speedway:

	Magnitude Optimal Int.	Magnitude Limits
Yerkes 3-inch photographic.	$6^M.6$	3–11
Yerkes 10-inch Bruce.	8.8	5–13
Yerkes reflector (24 inches)*.	11.0	7–15

* Determination by Professor G. Van Biesbroeck.

In the last column the magnitudes are given within which the reciprocity law holds, to a close approximation. Therefore, within this range, we would expect a gain of 1 mag. when the exposure time is increased 2.51 times. Outside of the tabular ranges this factor

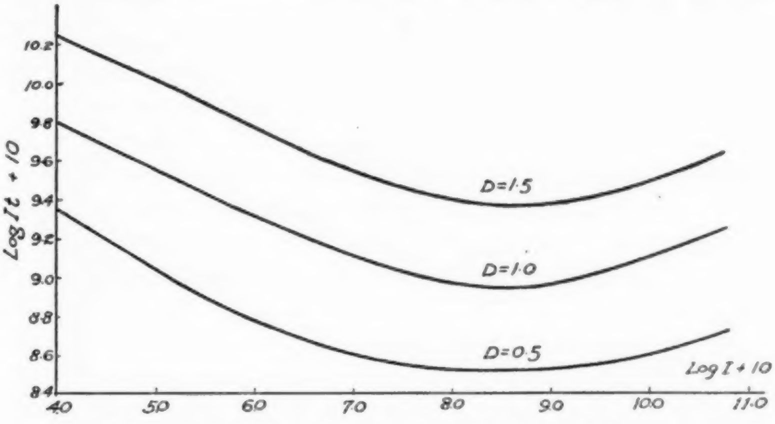

FIG. 1.—Reciprocity failure for Eastman Speedway plates

increases rapidly. It can be shown from Table I that if we go 3 mag. beyond the ranges given, the factor for exposure time becomes 3.5. The familiar falling off in the curve relating limiting magnitude to the logarithm of the exposure time is thus at least partly explained. The time factor for a gain of 1 mag. is usually considered to be 3.0. For exposures up to 10 minutes duration, it is found that the Yerkes 24-inch reflector gives a factor of 2.50. Beyond 10 minutes the factor increases at a rapid rate, which is in accord with the theory just outlined. The magnitudes in the last column appear, however, to need a correction of +2.

Effect of pre-exposure.—A great deal has been written on the effect of preliminary and simultaneous exposures in increasing the

apparent sensitivity of the photographic plate. The results have been contradictory. In blinking pairs of plates exposed respectively by Professor Barnard and myself, I have had opportunity to study the effect of a simultaneously acting fogging exposure of a varying degree of intensity. It has been claimed that an exposure of this kind is the most effective in giving an apparent increase of speed. Professor Barnard, as a rule, backed his plates, while I do not, except in special cases. A comparison of the relative intensity of the stars enmeshed in the halation ring in the case of the two plates should show the effect. The halation ring provides a wide range of intensities for study. In no case was the least difference noted. If the question is looked at from a theoretical standpoint, I think that it can be shown that in some cases the effect is present, while in other cases it is not. It should appear with certain types of emulsions in which the transition curve leading up to the straight-line portion of the curve of density plotted against exposure (not log exposure) is not insignificant. This transition curve varies notably with type of emulsion. I believe failure to detect any advantage is generally due to the fact that the plate with which comparison is made has already had a certain amount of exposure, either preliminary or simultaneous, enough to mask the effect. It is easy to show from the curve of density plotted against exposure that after a certain amount of pre-exposure no advantage can accrue. This curve must not be confused with the Hürtur and Driffield curve, or characteristic curve, in which the abscissa is taken as the logarithm of the exposure.

It is our custom to spread a backing over the glass side of the plate, to prevent halation.[1] It was noticed that when the backing did not entirely cover the plate the emulsion in front of the unbacked portion showed more field fog than over the adjacent backed areas. The effect increased with exposure time, being very marked on an exposure of ten hours. I do not know whether the phenomenon has been noted in the literature, or an explanation given. Experi-

[1] At the suggestion of Dr. C. E. K. Mees, director of the Kodak Research Laboratory, we are now using as a backing the commercial "Opaque," a yellow paste which only needs the addition of a little alcohol to make a very convenient and readily applied backing.

ments were made by the writer which proved that it could not be due to any direct or indirect action either of the backing or the plateholder. There was a possibility that the "Russell effect" might explain it.[1] To test this, a plate was backed over one-half of its area, placed in the plateholder, set aside in a dark closet for seventy-two hours, and then developed, along with a control plate. The fog, which was not excessive, covered the plate uniformly, proving that the backing was not responsible. Since the plateholders are made of duraluminum (aluminum is known to show the Russell effect strongly), a further test was made. Plates from the same box were placed in duraluminum and in brass plateholders, set aside for six days, then developed. Not a trace of fog, aside from a very light development fog, was noted on either plate. This plateholder is therefore perfectly safe.

The phenomenon cannot be due to general sky illumination, for, assuming that the opacity of the emulsion is 10, and that one-half of the light filtering through it is returned by total reflection, a generous allowance, it is seen that the direct fogging action of general sky illumination is twenty times the indirect action, with which we are concerned; in other words, the fogging action of the reflected light is insignificant compared with that caused by the direct illumination. There remains only starlight to consider. Roughly, the total light of the stars over an area of 75 square degrees of the sky, which corresponds to an area of 60 square inches on the plates taken with the Bruce 10-inch telescope, is equivalent to that of one first-magnitude star. It is quite probable that this amount of light returned by total reflection from the rear surface of an unbacked plate is sufficient to cause an appreciable amount of fog, on a prolonged exposure. For, an exposure with this instrument on Sirius of one-hour duration, with an unbacked plate, gave a dense halation ring about 3 inches in diameter. This ring without doubt contained enough silver to cause a very appreciable fogging over an area any-

[1] This is a fogging of photographic plates, discovered by Dr. W. J. Russell, due to hydrogen peroxide, a very powerful fogging agent. It has been shown that many metals and substances, notably paper, produce hydrogen peroxide to a greater or less degree by oxidation. It is of interest to note that the black border sometimes seen around fresh plates, and generally around plates which are old, is due principally to this cause, the wrapping paper being the active agent.

where from 6 to 12 inches in diameter. It would be very difficult to obtain quantitative data on the subject, but it is quite evident that we have a *vera causa*.

If the photographic action of the general sky illumination should be of the same order of magnitude as starlight, the foregoing effect would be masked and rendered insensible. Van Rhijn and others have shown that, visually measured, the intensity of the general sky illumination is several times that of starlight, If, however, this light is auroral, as seems highly probable, its photographic intensity would be small, and should not mask the action of the starlight returned from the back of the plate. Whatever the explanation may be, there is no doubt of the reality of the phenomenon, nor of the necessity of backing plates, not merely to prevent halation, but for the purpose of reducing general field fog, on long exposures.

Method of reproduction.—The faint nebulae composing the outlying nebulosities of the Pleiades may be designated as of the second order of faintness. It accordingly requires very special treatment, with a good measure of patience to set forth on paper the delicate beauty residing in their halftones and in their filmy details of structure. Professor Barnard and others have photographically reproduced the stronger condensations of the nebulosity. Barnard was able to see on his negatives more than he was able to render photographically, by the methods of reproduction then available. In collaboration with Mr. E. Calvert, he made a drawing of all the detail that he was able to trace on his best negatives—all told, an imposing amount. It appears to be true that the earlier workers could see more on their negatives than they were able to reproduce. I have succeeded in showing that the methods now available enable one to reproduce more than he can see. The factor of importance in this connection is the kind of photographic plate used in the reproduction process. I am using a process panchromatic plate, working behind a red filter. While this combination is known to give the maximum of contrast, the filter alone accounting for a factor of 2, I am not convinced that it is entirely a question of contrast. My experiments suggest that success in reproducing the faint detail that lie on the plate on the threshold of visibility depends on yet another factor, namely, the degree or fulness of development of the original

negative. I have had no success in applying the method to negatives which had "soft" or light development. Since negatives of the planets, exposed and developed for surface detail, are as a rule developed "soft," necessary on account of the great range of light-intensity over the planetary surface, it is not to be expected that the technique of planetary photography can be improved in this way.

In general, in the reproduction of nebular details, it is necessary to go to a third negative, making in all four transfer processes. For all of these transfers I have used the method of contact printing. This has the advantage of even illumination, a very important factor. If transfer by projection is resorted to, the lens used should be unusually well corrected, in order not to degrade the very fine images near the corners of the plate. The principal difficulty arising from the use of this method is due to lack of uniformity of sensitivity of the emulsion, either on the original negative or on any one of the transfer plates. The advantages, however, obviously more than offset this disadvantage, which is completely discounted by duplicating the original negative, as well as the transfer processes. In the case of very long exposures the advantage of twin cameras is obvious. In reproducing the Barnard negative shown in Plate VIII, our experience was instructive. The third negative showed a nebulous region, which excited suspicion as to its reality. It was traced back to the first positive, on which it was exceedingly faint. Not a trace of it, however, could be seen on the original negative, either by Miss Calvert or myself. Repetition of the transfer plates definitely proved its existence on the original negative, thus demonstrating the dictum noted above that we can reproduce what we cannot see. This is perfectly obvious on general principles, since visibility is a matter of contrast.

It may well be asked, therefore, why the enhanced contrast does not bring out more faint stars, which doubtless have left some impression on the plate, but lie beyond the threshold of perception on the original negative The reason for the failure is, I think, due to the development of the factor of graininess in the reproducing process, which progressively increases as the contrast is "stepped-up." Graininess, when small areas or star images are viewed, is a

serious defect. In viewing extended areas, such as nebulosities, it is not detrimental.

A further difficulty in producing what may be called "high-power" copies, in addition to the lack of uniformity of the emulsion, arises from the vignetting action of the lens, a defect inherent in all anastigmats due to their length, so that the relatively small difference of intensity between center and border of the original plate becomes magnified to a ruinous degree in copying. This inequality is corrected in the first positive by the following device, which applies the principle of the vignetting lens in the opposite direction. A small round illuminated area used as printing light is placed in a cylinder of such length and opening as to give the requisite falling off in illumination at the distance of the printing frame. Adjustment can be made either by calculation or by trial.

PLATE VI. MILKY WAY IN MONOCEROS

Plate VI is centered on the nebulous region of 15 Monocerotis, $\alpha = 6^h\ 36^m$, $\delta = +10°$ (1927.0), the galactic latitude being $+4°$. The limits in right ascension are 5^h48^m and 7^h28^m, and in declination $0°$ and $+20°$. It was exposed on March 3 and 4, 1927, with a total exposure of eight hours, four hours on each night, and was reproduced as explained on page 291. Mr. A. Pogo assisted in the exposure. The brighter nebulae, including the very beautiful one N.G.C. 2237 below the center of the plate, are, of course, very much overexposed. The dark nebula to the north of 15 Monoceros can be traced as far as Gamma Geminorum, the bright star seen near the northern edge of the plate. This plate is of interest in showing the great extent of the bright nebulosities in the Milky Way. In width its limits appear to be well defined. While it is doubtful if it extends in a southerly direction below N.G.C. 2237, its northern limit appears to be beyond the range of the plate. For another study of this region, see Barnard's Plates 28 and 29 in *Publications of the Lick Observatory*, Volume **11**. The scale of the plate is: $1° = 9.3$ mm.

PLATE VII. NEBULOSITIES IN TAURUS AND PERSEUS

In April, 1927, when the photovisual lens was added to the mounting, the camera was rotated 90°, making the long side of

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$\alpha = 3^h 42^m, \quad \delta = + 24^\circ$

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AU US AND PERSEUS

Scale: 1° = 0.93 cm

the plate parallel to the hour circles. This change was necessary, on account of the limited width of the opening in the dome and the wide angle of view of the cameras. Plate VII was taken on January 21 and 22, 1928, with a total exposure of ten hours—five hours on each night. The limits in right ascension are 3^h4^m and 4^h24^m, in declination $+12°$ and $+37°$. Mr. Y. C. Chang assisted in the guiding. In the reproduction, the inner nebulosities and stars of the Pleiades are necessarily blotted out. The nebula at the northern edge of the plate is N.G.C. 1499, discovered visually by Barnard in 1885. A portion of the dark markings in Taurus and the accompanying nebulous background are seen at the east-central portion. It is noteworthy that the nebulosity encompassing the Pleiades does not seem to be a part of the great nebulosity in Taurus and in Perseus, the latter of which is seen below and to the west of N.G.C. 1499. There is a broad, dark lane to the east of the Pleiades, suggesting obscuration by an extended dark nebula. The division between the Pleiades and the nebulosity in Perseus does not appear to be of this nature. One gets the impression that these nebulae are distinct. The bright and dark nebulae in Perseus have been extensively studied by Barnard.[1] Due, I think, to plate defects, the northwestern part of the bright nebulosity of Perseus has been curtailed, in the present plate, the nebulosity doubtless extending farther than shown. Another exposure is planned for next winter, centered on Omicron Persei, which should show the true form and extent of this nebula, and decide in addition whether N.G.C. 1499 is involved in it. In general, the regions near the edge of the plate are difficult to reproduce, so that one should be cautious in drawing conclusions as to the existence of these extremely faint objects which lie far from the center of the plate.

Perhaps one of the most remarkable features on the plate, although not obtrusive, is the system of parallel curved markings, extending east and west, below the center. I am equally divided in my opinion as to their reality. It will be noted that they fall in line with the undoubted east-west curved nebulosities which form the outlying nebulosity of the Pleiades, and this therefore gives them a suggestion of reality. I hope to make an appropriately centered

[1] *Atlas of Selected Regions of the Milky Way*, Plate 3.

exposure next winter which will settle the matter. This also should give the true outlines of the apparently extensive nebulosity southwest of the Pleiades, which no doubt forms part of the mass of nebulosity involving the Pleiades.

The main feature of the outer nebulosities of the Pleiades proper are the two bright nebulous streamers, running east from the Pleiades, about 3° apart, each showing a great deal of detail. In addition to this, almost completely encircling the group, are a great number of small, bright detached masses of nebulosity, the largest of which lies about 3° (28 mm) southwest of the center of the Pleiades.

PLATE VIII. THE OUTER NEBULOSITIES OF THE PLEIADES

This plate has been reproduced by the writer from an original negative taken by Professor Barnard at Mount Wilson on September 7, 1905, with the Bruce 10-inch lens, with an exposure of 3^h48^m. The negative is a particularly fine one, and shows beautifully the outlying nebulosities in the immediate vicinity of the Pleiades group. Between the two curved nebulous arms extending eastward, suggesting the outstretched arms of a crab, the Pleiades forming the body, numerous small detached masses of nebulosity are faintly outlined. This is better seen in the third transfer negative, from which the print was made, features of this delicacy losing in transferring to paper. On account of the larger scale, the small nebulosities are better shown than on Plate VII. In this comparison the latter suffers from the fact that it shows more of the general uniform nebulosity in which the Pleiades are enmeshed, thus degrading the contrast. In particular, note the nebulosity directly west of the Pleiades, of which there is no indication on this plate. The difference between the two plates in the region west of the Pleiades is a matter for further investigation.

Concerning the nebulosity surrounding the Pleiades, Barnard wrote:

For many years during my comet seeking I have known of a vast and extensive but very diffuse nebulosity north of the Pleiades. Other masses of this diffused matter make their presence known by a general dulling of the field when sweeping in the region of the cluster. . . . . To the north of the Pleiades, from

PLATE VIII

West

OUTER NEBULOSITIES OF THE PLEIADES

Scale: 1° = 1.79 cm

From a negative by the late E. E. Barnard

$\alpha = 3^h 20^m$ to 4 hours and beyond, and from $\delta = 30°$ to several degrees farther north, is a region singularly devoid of small stars but covered with large masses of very diffuse nebulosity; this part of the sky will attract the attention of any one in sweeping over it with a very low power on an ordinary telescope. The field is dull with feeble luminosity.[1]

This description refers to the nebulosity in Perseus, shown in the upper part of Plate VIII, and is not to be confused with the outlying nebulosities of the Pleiades proper. Since this pioneer work of Barnard's in the detection visually of extended nebulosities, a notable amount of work has been done in the same field by J. G. Hagen, whose maps of numerous other regions of obscuration should prove useful in the present photographic survey.

So far as I am aware, Barnard was the first to photograph the outlying nebulosities of the Pleiades, at the Lick Observatory, on December 6 and 8, 1893,[2] the exposure time being $10^h 15^m$. It is of historical interest to note that Isaac Roberts, in 1897, was unable to obtain a trace of these nebulosities, either with his 20-inch reflector or 5-inch Cooke portrait lens, exposed for ten hours under excellent conditions. He wrote, "The six photographs . . . . entirely discredit the existence of Professor Barnard's nebulosity."[3]

YERKES OBSERVATORY
March 6, 1928

[1] *Astronomische Nachrichten*, **136**, 193, 1894.

[2] *Publications of the Lick Observatory*, **11**, Plate 15, 1913.

[3] *Monthly Notices*, **58**, 394, 1898.

A COMMENT ON CERTAIN EQUATIONS IN THE THEORY OF RADIATIVE EQUILIBRIUM

By K. P. WILLIAMS

ABSTRACT

An examination is made by means of the *theorem of mean value* of a series of equations that occur in Eddington's *Theory of Radiative Equilibrium*. The *result disagrees* with Eddington's conclusion. A form for the *variation of temperature and density* within a star is assumed, which allows the equations to be integrated, and results that confirm the general discussion are obtained.

1. In his work, *The Internal Constitution of the Stars*, Eddington makes a digression to explain and lament the shortcomings of mathematicians (pp. 101–103). No one will deny that an effort to carry into physical theories many of the refinements of modern rigorous analysis is likely to lead to confusion, and is more likely to impede progress than to assist it. Eddington's remarks are interesting and stimulating, and mathematicians may well reflect upon the caution that they contain. However, Eddington not only describes how awkwardly the class behaves when adventuring away from home, but he indicates that their deplorable habits in public are to be attributed to the fact that they are at least queer when in their own houses. In his analysis of the aim of mathematics Eddington is in error. To claim that the end of mathematics is to construct proofs is like saying that the end of physics is to construct better and longer meter sticks. "Insight"—to use a word highly favored in the pages referred to—is the end of mathematics quite as much as of physics. But the mathematician requires that his intuition and his conjectures be replaced ultimately by convincing proof. Until he has done that he is not sure but that what he thought was insight was mere deception. Eddington does concede that a modicum of the world's precious supply of "insight" is in the hands or the heads of the mathematicians; but they degrade it. They use their small part of the divine gift in the base occupation of constructing proofs. To make a good proof is indeed no easy thing, and it may even require intelligence; quite as much as it requires intelligence as well as

dexterity to put together new apparatus in order to demonstrate one's suspicions about electrons. It is not likely that Eddington's claim that a holy spirit of insight hovers over the equations that the physicist writes and absolves them from blame for minor delinquencies and lapses will prevent mathematicians from laying seemingly unhallowed hands upon them.

It may well be that mathematicians should stay without while a new physical theory is getting itself born. But after its survival is assured and a certain robustness has been attained, they may well be admitted not only to congratulate the parents for their achievement, but to be allowed some freedom of examination. As a result of their pokings the new theory may become not only more rounded and comely, but even more sturdy. Themselves unable to fecundate, they may nevertheless be wholesome companions for the offspring of those who can.

It seems probable that the presence in a physical theory of a large amount of mathematical analysis tends to make it imposing, and gives conviction as to its certainty. The one means to prevent equations from being a mere deception is to give them a careful scrutiny. In its final form it would appear that a physical theory should achieve something more than trustworthy results. It deserves to have all irrelevancies and ambiguities removed. Under the goad of a purely mathematical examination that may seem to come near to carping criticism, the physicist may be stimulated into finding methods of deduction free of objection. Although the final result may not have been altered, his theory has been improved.

This rather long introduction seemed necessary since the equations it is proposed to examine here occur immediately after Eddington's digressive remarks. It should be said that the note was written before the appearance of the book, and was based upon the article, "Das Strahlungsgleichgewicht der Sterne," in Volume **7** of the *Zeitscrift für Physik*, which seemed to be a definitive form of Eddington's theory on radiative equilibrium. The more recent work appears to present the development in quite the same way. On that account the paper has been left as it was. References are to the *Zeitscrift* article.

2. Denote by $I(\theta)d\omega$ the energy that crosses in one second a

square centimeter lying within a cone of solid angle $d\omega$, which makes an angle θ with the normal to a sphere of radius r within a star. Eddington writes

$$I(\theta)=A+B\cos\theta+CP_2(\cos\theta)+DP_3(\cos\theta)\ \ldots\ ,$$

where P_2, P_3, ... , are the Legendre polynomials, and A, B, C, ... , are functions of r to be determined. If one is not near the surface of the star it seems that the flow of energy will be sensibly independent of direction; which indicates that B, C, D, ... , are small in comparison with A. But before the coefficients are to be rejected an effort is to be made to determine their relative magnitudes.

3. Let k be the mass coefficient of absorption, ρ the density, μ a universal constant that relates the emission of energy to the fourth power of the temperature, and ϵ a constant to care for other than temperature sources of energy. By considering what takes place within a small volume the following equations are arrived at (p. 355):[1]

$$\frac{1}{3}\left(\frac{dB}{dr}\right)+\frac{2}{3}\frac{B}{r}=k\rho(\mu T^4-A)+\frac{\epsilon\rho}{4\pi}\ , \tag{1}$$

$$\frac{dA}{dr}+\left(\frac{2}{5}\frac{dC}{dr}+\frac{6}{5}\frac{C}{r}\right)=-k\rho B\ , \tag{2}$$

$$\left(\frac{2}{3}\frac{dB}{dr}-\frac{2}{3}\frac{B}{r}\right)+\left(\frac{3}{7}\frac{dD}{dr}+\frac{12}{7}\frac{D}{r}\right)=-k\rho C\ , \tag{3}$$

$$\left(\frac{3}{5}\frac{dC}{dr}-\frac{6}{5}\frac{C}{r}\right)+\left(\frac{4}{9}\frac{dE}{dr}+\frac{20}{9}\frac{E}{r}\right)=-k\rho D\ , \tag{4}$$

. . . .

In the equations actually given by Eddington the terms in (4) that involve E are omitted.

It is to a discussion of these equations that the present note is devoted. The discussion that Eddington gives is very brief, and leads

[1] In Eddington's book the equations are given on p. 106. The only change is that the right-hand member of (1) is different, the temperature and other radiation not being separated.

him to the conclusion that $A:B:C: \ldots .$, as $1:10^{-10}:10^{-20}: \ldots .$ No attempt is made by him to integrate the equations. The only objection that can be raised to the latter plan would seem to be its difficulty. So long as k and ρ are unspecified, actual integration is impossible. But since the left members are simple, some knowledge can be obtained by employing the theorem of the mean value. It should at least furnish us with some idea concerning the relative orders of the functions defined by the equations.

Equation (4) can be written

$$\frac{dE}{dr}+\frac{5}{r}E=-\frac{9}{4}k\rho D-\frac{9}{4}\left[\frac{3}{5}\frac{dC}{dr}-\frac{6}{5}\frac{C}{r}\right],$$

from which we find

$$E=\frac{1}{r^5}\left[-\frac{9}{4}\int_0^r k\rho Dr^5\,dr-\frac{27}{20}\int_0^r r^5\frac{dC}{dr}\,dr+\frac{54}{20}\int_0^r Cr^4dr\right].$$

But we can write

$$\int_0^r k\rho Dr^5dr=rr_1^5(k\rho D)_{r=r_1},$$

where $0<r<r_1$. Similarly,

$$\int_0^r r^5\frac{dC}{dr}\,dr=r^5C-5\int_0^r Cr^4dr,$$

so that

$$-\frac{27}{20}\int_0^r r^5\frac{dC}{dr}\,dr+\frac{54}{20}\int_0^r Cr^4dr=-\frac{27}{20}r^5C+\frac{189}{20}\int_0^r Cr^4dr=$$

$$-\frac{27}{20}r^5C+\frac{189}{20}rr_2^4(C)_{r=r_2},$$

where $0<r_2<r$.

Therefore

$$E=-\frac{27}{20}C-\frac{9}{4}\frac{r_1^5}{r^4}(k\rho D)_{r=r_1}+\frac{189}{20}\frac{r_2^4}{r^4}(C)_{r=r_2}.$$

This equation suggests that E is of equal or greater order than C. It is true that r_1 and r_2 are unknown, but it is difficult to see how they could have values such that the result above would agree with Eddington's conclusion.

In a similar way we obtain from (2)

$$C=-\frac{5}{2}A-\frac{5}{2}\frac{r_3^3}{r^2}(k\rho B)_{r=r_3}+\frac{15}{2}\frac{r_4^2}{r^2}(A)_{r=r_4}\,,$$

where $0<r_3<r$, $0<r_4<r$. This equation indicates that C is of at least the same order as A. Combining the results, it appears that a coefficient even as advanced as E, which Eddington neglects entirely, is of at least the same order as A.

4. A reasonable procedure now is to assume forms for T and ρ that will allow an actual integration of the equations. The manner in which T and ρ vary within a star are, of course, not known, but if we assume something not totally beyond possibility, we can expect that the behavior of A, B, C, ... , will be something like what it is in actuality.

It will first be shown, following Eddington, that (1) can be replaced by two equations.

We begin by noting that the amount of energy that flows out through a sphere of radius r is $\frac{16}{3}\pi^2 Br^2$, irrespective of any hypothesis concerning C, D, ... This remark is made by Eddington, though without detailed proof (p. 357).[1] The excess of energy that flows out of a spherical shell of thickness dr over that which flows in is therefore

$$\frac{d}{dr}\left(\frac{16}{3}\pi^2 Br^2\right)dr\,.$$

In the stationary state this must equal the energy originating within the shell, which is (Eddington, p. 357):

$$4\pi r^2 dr\cdot\rho\epsilon\,.$$

Therefore

$$\frac{d}{dr}\left(\frac{16}{3}\pi^2 Br^2\right)=4\pi r^2\rho\epsilon\,,$$

[1] For the sake of completeness a proof will be given at the end.

from which we find

$$\frac{1}{3}\frac{dB}{dr}+\frac{2}{3}\frac{B}{r}=\frac{\epsilon\rho}{4\pi}\,. \tag{5}$$

It follows from (1) that

$$A=\mu T^4\,. \tag{6}$$

We may therefore replace (1) by (5) and (6).

Before proceeding to an integrable case, it is desirable to give the values of certain of the constants. We have (Eddington, p. 358):

$$\mu=\frac{ac}{4\pi}$$

where

$$a=7.06\times 10^{-15}\,,\qquad c=3\times 10^{10}\qquad \text{(C.G.S.)}\,.$$

Hence

$$\mu=1.68\times 10^{-5}\,.$$

We shall also assume that k and ϵ are constants. Remarks on such assumptions are made by Eddington (pp. 363, 364). We shall also put $\epsilon_0=\epsilon/4\pi$.

5. We shall now assume the following forms for T and ρ:

$$T=T_0e^{-r/b}\,,\qquad \rho=\rho_0e^{-r/c}\,, \tag{7}$$

where T_0 and ρ_0 are the central temperature and density, respectively, and b and c are to be determined so that the formulae give values agreeing with those near the surface. Such a manner of decrease of temperature and density is not contradicted by any actual knowledges of conditions within a star. The results to which they lead should be at least suggestive as to the relative orders of the functions $A, B, C, \ldots$.

When equation (5) is integrated and the constant determined so that $B=0$ when $r=0$, we find

$$r^2B=-3\epsilon_0\rho_0c^3e^{-r/c}[r^2/c^2+2r/c+2]+6\epsilon_0\rho_0c^3\,. \tag{8}$$

We next insert in (2) the value of B from (8) and that of A obtained from (6) and (7). Integrating the result we find

$$\left.\begin{aligned} r^3C=&-\frac{5\mu T_0^4}{2}b^3\left[\frac{r^3}{b^3}+\frac{3}{4}\frac{r^2}{b^2}+\frac{3}{8}\frac{r}{b}+\frac{3}{32}\right]e^{-4r/b}\\ &-\frac{15}{2}\epsilon_0\rho_0^2kc^5\left[\frac{1}{2}\frac{r^3}{c^3}+\frac{7}{4}\frac{r^2}{c^2}+\frac{11}{4}\frac{r}{c}+\frac{11}{8}\right]e^{-2r/c}\\ &+15\epsilon_0\rho_0^2kc^5\left[\frac{r}{c}+1\right]e^{-r/c}\\ &+\frac{15\mu T_0^4}{64}b^3+\frac{75}{16}\epsilon_0\rho_0^2c^5k\ , \end{aligned}\right\} \tag{9}$$

the constant being again chosen so that $C=0$ when $r=0$.

6. We shall next make the moderate assumption that $T_0=10^6$. Let the radius of the star be $R=10^{12}$, and suppose that $T=10^4$ for $r=R$. Then from (7)

$$b=\frac{R}{2\log_e 10}=\frac{\log_{10}e}{2}R=.2171R\ .$$

We shall take the approximate value $b=R/5$.

Suppose that $\rho=\rho_0/e^3$ when $r=R$. The particular value is assumed to simplify calculation. The quantity e^3 could be replaced by e^4 or any comparable value without essentially altering the result.

From (7) it follows that $c=R/3$.

We next calculate the values of B and C for $r=R/4$. For this value we have

$$\frac{r}{b}=\frac{5}{4}\ ,\qquad \frac{r}{c}=\frac{3}{4}\ .$$

From (8) we find

$$r^2B=.012\times\epsilon_0\rho_0R^3\ ,$$

and from (9)

$$r^3C=.015\times\mu T_0^4R^3+.038\times\epsilon_0\rho_0^2kR^5\ .$$

Therefore

$$r\frac{C}{B}=\frac{\mu T_0^4}{\epsilon_0\rho_0}+3\rho_0kR^2\ ,$$

or, since $r=R/4$,

$$\frac{C}{B}=\frac{4\mu T_0^4}{\epsilon_0\rho_0 R}+12\rho_0 kR\ .$$

Using the values of μ, T_0, and R, we find

$$\frac{C}{B}=\frac{6.72\times 10^7}{\epsilon_0\rho_0}+12\rho_0 k\times 10^{12}\ .$$

The ratio C/B is evidently large for any acceptable values of k, ϵ_0, ρ_0. The result therefore exhibits an incompatibility between the following: (*a*) the assumption that each coefficient in the expansion in § 2 is of much smaller order than the preceding, (*b*) the correctness of equations (1)–(6) and thus the soundness of the reasoning that underlies them, (*c*) the assumption that temperature and density decrease in a way even remotely similar to that given in (7).

7. It should be observed that in this investigation all real mathematical niceties have been ignored. If it be said that equations (1)–(4) are only approximations to actual conditions, it would seem that the general results arrived at in § 3 have at least the same degree of reliability. One is therefore led to question whether equations (1)–(4) have any actual significance in the theory. It might be said that they occur in Eddington's book as affording the basis for an alternative and more convincing derivation of one of the fundamental equations of the theory. The investigation of the special case confirms the results in § 3.

8. NOTE.—To prove the fact referred to in section 4 we proceed as follows:

Let E_0 denote the total radiation outward that passes through a unit area, then

$$E_0=\int_0^{2\pi}d\vartheta\int_0^{\pi/2}I(\theta)\cos\theta\sin\theta\, d\theta\ ,$$

the factor $\sin\theta d\theta d\vartheta$ denoting the surface element, and $\cos\theta$ being introduced since radiation is considered in a direction making an angle θ with the normal to the surface.

Letting $z=\cos\theta$, we have

$$E_o = 2\pi\int_0^1 [A+Bz+CP_2(z)+DP_3(z)+\ldots.]\,zdz$$

$$=\pi\left[A+\frac{2}{3}B+a_2C+a_4E+\ldots.\right],$$

where a_2, a_4, ... , are constants, since

$$\int_0^1 zP_{2n+1}(z)dz=0\ .$$

Let E_i denote the total radiation inward through the element. Then

$$E_i=-\int_0^{2\pi}d\vartheta\int_{\pi/2}^{\pi}I(\theta)\cos\theta\sin\theta\,d\theta$$

$$=\pi\left[A-\frac{2}{3}B+a_2C+a_4E+\ldots..\right].$$

Hence

$$E_o-E_i=\frac{4}{3}\pi B\ .$$

The total flux outward through a sphere of radius r is therefore $\frac{16}{3}\pi^2Br^2$.

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WAVE-LENGTHS IN STELLAR SPECTRA OF CLASS B

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ABSTRACT

Wave-lengths in classes Bo–B8.—Published and unpublished *measures* of three-prism spectrograms of *helium stars* by Frost and Adams were re-reduced. Clark's wave-lengths for *O* II and *N* II, Barrell's values for *Si* III, and Merrill's wave-lengths for the *He* pair at λ 4713, blended according to the intensities of the components, were substituted for the normal wave-lengths originally employed. For the *titanium comparison lines* the recent wave-lengths of Brown and Crew were used. Individual stellar wave-lengths were determined on the international system for 58 lines between λ 4317 and λ 4713, for each of 24 stars. The total number of separate plate measurements was 231 on 174 spectrograms. For the means for classes Bo–B8 the probable errors for lines represented by numerous measures range from ±0.002 to ±0.007 A. The probable error for an observation of unit weight is ±0.052 A. A slight effect due either to peculiarities of certain comparison lines or to coma is indicated in the measures.

The *revised means of the radial velocities* for the individual stars are given.

Discussion.—The agreement between the means of observed wave-lengths and their normal values is, on the whole, very good. For a few lines moderate deviations occur, which are attributed to inaccuracies in the normal wave-lengths, due, in some instances, to appreciable accidental errors and in others to the arbitrary blending of close pairs according to the intensities of the components. Possibilities are discussed for improving the normal wave-lengths of blends. A systematic difference of 0.013 A between the observed wave-lengths for *O* II and *N* II relative to those for *H*, *He*, *Mg* II, and *Si* III as a group may be due to systematic errors in the normal wave-lengths or to relative motions between the two groups of gases. Decision between the two alternatives will require additional study of laboratory wave-lengths. Owing to the direct bearing which stellar wave-lengths have upon several astrophysical problems of a fundamental nature, *laboratory wave-lengths should be employed* as normal values in preference to adjusted stellar wave-lengths, except in special cases and for special purposes.

The statement that of all wave-lengths published on the Rowland system the only ones that still have any appreciable value are those given in Rowland's "Preliminary Table of Solar Spectrum Wave-Lengths" expresses the fairly general sentiment among physicists who are engaged in the determination of wave-lengths. The reasons for this lie in the more recent discovery that it is necessary to adopt definite standard experimental conditions in determinations of wave-length, due to various disturbing influences, such as pressure-shifts, pole-effect, etc., which previously had been either unknown or imperfectly understood. The foregoing statement seems valid for essentially all laboratory determinations of wave-length on the Rowland system.

In view of this fact, observers of stellar radial velocities and

wave-lengths have for some time been anxious to discontinue the use of the Rowland system. A great quantity of very valuable stellar work will suffer the same fate that has befallen the laboratory results unless special efforts are made to redeem it. The circumstances in the two cases are to a large extent distinct. In the laboratory the experimental conditions are subject to the control of the observer. For most, if not all, of the early laboratory spectrograms the records are incomplete in regard to the exact experimental conditions, thus rendering the results obtained from these spectra worthless where the highest accuracy is desired. In stellar work only the reference spectra may be thus controlled, the physical conditions giving rise to the stellar spectra being entirely beyond the reach of manipulation by the observer. Thus, stellar spectra, photographed with a properly adjusted spectrograph and with good comparison spectra, have intrinsically a permanent value comparable to that of laboratory spectra obtained under standard conditions.

The aim of this paper is to begin the redemption of some of the early results which are based on the Rowland system. The use of the international system of wave-lengths for B stars is now quite practicable, because good—though not necessarily definitive—laboratory wave-lengths have recently become available for the lines occurring in the spectra of these stars. Below are given, in condensed form, the results of an endeavor to obtain wave-lengths from the very valuable measures of B stars by E. B. Frost and W. S. Adams contained in Volume 2 of the *Publications of the Yerkes Observatory*, supplemented by unpublished measures.

A large proportion of B stars show a variable radial velocity, and some of them are known to undergo fluctuations of light. Some have a variable range of velocity variation and even exhibit different variations for different lines or group of lines. Only a mere beginning has been made in the study of variations in the spectra which accompany the regular and irregular fluctuations of light and of velocity. Accordingly, it would be highly desirable to publish the individual observed wave-lengths for each spectrogram so as readily to make possible a satisfactory revision of the results whenever new information in regard to the normal wave-lengths, and, to a less

extent, in regard to the reference wave-lengths, shall become available. It has not proved practicable to give such details because of the high cost of publication. However, in order that these results may nevertheless contribute in a fundamental way toward various astrophysical problems it is the intention later to publish the individual observed wave-lengths in a cheaper form for separate distribution.

The published measures of Frost and Adams referred to above are almost unique among results for radial velocity in giving sufficient data for a complete rediscussion, and for this reason their value is of a much more extensive and more permanent nature than usual. The micrometer settings are, of course, the essential data which would suffice for a complete reduction, *ab initio*, by either the same or independent methods. In the present instance, the original reductions were utilized as far as was practicable. Professor Frost kindly placed the original measuring sheets at my disposal, thereby greatly facilitating the work. Although the various numerical quantities involved had not been calculated in duplicate, a fairly good check on these reductions was obtained by a comparison of the dispersion corrections for star lines with the computed corrections for the comparison lines, by a close inspection of the individual displacements and velocities, and by a more detailed revision of cases where residuals departed considerably from the means. In this way a number of accidental errors in the original reductions were found and eliminated. Each step was checked so as to reduce to a minimum the chance of introducing new errors.

The most important change introduced was a revised list of normal wave-lengths for the stellar lines. The wave-lengths for enhanced oxygen and nitrogen by J. S. Clark and A. Fowler[1] and for doubly enhanced silicon by H. Barrell,[2] all on the international system, were substituted for the corresponding normal values employed by Frost and Adams. Also, for the helium doublet at 4713 the blend of P. W. Merrill's wave-lengths combined in the usual

[1] *Astrophysical Journal*, **40**, 332, 1914, and *Proceedings of the Royal Society*, A, **110**, 476, 1926.

[2] *Monthly Notices, R.A.S.*, **83**, 322, 1923. See also *Astrophysical Journal*, **55**, 363, 1922, and **57**, 58, 1923.

manner according to intensities was adopted. The normal wave-lengths which were employed in the revised reductions are given in the tables.

Laboratory wave-lengths should, of course, be used for the reference spectra. Rowland's wave-lengths which, according to common practice, had been employed for the titanium comparison lines, are solar wave-lengths and are therefore not in every instance due to titanium alone. Accordingly, the laboratory wave-lengths for titanium by F. L. Brown[1] and H. Crew,[2] on the international system, were adopted, and corrections based on separate adjustments for each plate were applied to the results.

As an additional and independent check on the various steps for the conversion from the Rowland to the international system, the measures for twenty-eight spectrograms were actually re-reduced, using only the published micrometer settings and entirely new reduction tables based directly on the international system of wave-lengths. Very good agreement was found between the results derived from these *ab initio* reductions and those which had been previously obtained for the same plates by the revision of former reductions as outlined above.

Although the results represent the international system in so far as this is controlled by methods of reduction, there may still remain slight effects which are inherent in differences in quality and intensity among the comparison lines. Thus, supplementary tests indicated that the titanium comparison line 4395 produces distinctly high wave-lengths for a stellar line based directly upon this reference line. According to Frost, 4395, which is enhanced in the spark, is too broad for the best settings; 4399 is one of the very best lines on a plate correctly exposed for the comparison spectrum, while 4387 is usually too weak. Professor Frost informs me that he has made a number of attempts to eliminate effects due to physiological difference in making settings on a very black and broad line as compared with a fainter, more or less grayish line, on which the thread—or line ruled on glass—is better visible than on the very black line. Double threads were also employed by him without the expected improvement. The inference to be drawn from these considerations

[1] *Astrophysical Journal*, **56**, 53, 1922. [2] *Ibid.*, **60**, 108, 1924.

is that in future measurements the line 4395 should be either entirely omitted or at least be not made the sole basis of reference in this region of spectrum. The line 4387 is generally somewhat weak. Both 4387 and 4399 could to advantage be included in the measures. Although results obtained from individual measures may be somewhat affected from these causes, in the means derived from several plates such effects are undoubtedly in part smoothed out and reduced to slight residual effects by a considerable range from plate to plate in the intensity of the comparison spectra.

Tables I and II contain the means of the observed wave-lengths for each star. The wave-lengths given at the head of the columns in Table I are the normal values which were employed in the reductions. These taken collectively constitute the system upon which the observed wave-lengths are based. The given weights are the sums of weights assigned by Frost and Adams at the times of measurement. Table II is supplementary to Table I and contains the observed wave-lengths for the lines which have a total of less than seven measures. The accuracy of the final means given at the foot of Table I is represented, probably fairly closely, especially for the lines having considerable weight, by the computed probable errors. The probable error of a mean for the lines having more than one hundred measures was found on the average to be ± 0.0031 A, while for the lines represented by between fifty and one hundred measures it is ± 0.0053 A. In view of the inferior quality of many of the lines this is satisfactory.

An inspection of Table I shows that the observed and normal wave-lengths differ from each other in varying degree for the different lines. In fact, these wave-lengths furnish valuable evidence on the directions along which additional verification of laboratory results is needed and on the relative systematic relationships in stellar spectra of Class B of the wave-lengths for the different elements or groups of elements. Their value toward the latter problem will be enhanced with the solution of the former.

Only small systematic discrepancies for different groups of elements are revealed in Table III. The weighted mean of all is practically zero, as it should be. Relative to the mean for all lines the observed wave-lengths for enhanced oxygen and nitrogen are on

TABLE I

WAVE-LENGTHS, ON THE INTERNATIONAL SYSTEM, OBSERVED WITH THREE PRISMS

STAR	SP. CLASS	4340.466 $H\gamma$		4345.570 O II		4347.429 O II		4349.434 O II		4351.275 O II		4366.906 O II		4387.931 *He*		4414.888 O II		4416.974 O II		4437.548 *He*		4447.035 N I O II		4471.505 *He*	
		Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.
ε Ori..	B0	.539	6	....	....	....	....	....	....	....	....	....	....	.999	8½	....	....	....	....	....	....	....	....	.497	14
ζ Ori..	B0	.501	9	....	....	....	....	....	....	....	....	.871	2	.942	8	.933	1	.888	2	.554	4	....	....	.483	12
κ Ori..	B0	.500	7	....	....	....	....	.475	1	.256	1	.862	2	.903	9	.871	2	.046	1	.582	3	....	....	.510	22½
ζ Per..	B1	.418	4	.586	2	....	....	.365	2	....	....	.911	3	.932	6½	.889	8	.008	5	....	....	....	....	.515	11
η Ori..	B1	.446	11	.560	12	.417	5	.407	32	.297	6	.845	26	.957	30	.911	56	.977	31	.560	11	.987	1	.537	66
β CMa	B1	.442	1½	.526	1	....	....	.399	1½	.309	1	.850	3	.956	5	.901	11	.984	5½	.590	1	....	....	.540	9
ε CMa	B1	.434	3	.503	3	....	....	.439	2	....	....	.878	3½	.954	4½	.879	6½	.971	6	.588	4	.953	1	.549	10
β Cep.	B1	.416	7	.653	3	.384	5	.402	18	.320	8	.840	23½	.943	31½	.889	47½	.976	32	.546	14	.907	1	.539	56
γ Peg..	B2	.464	7	....	....	....	....	.403	1	....	....	.864	1	.974	16	.897	22	.997	9	.571	19½	....	....	.512	26½
ζ Cas..	B2	.464	8	....	....	....	....	....	....	....	....	.859	½	.958	14	.839	2½	.969	1	.566	8½	....	....	.498	14
δ Cet .	B2	.498	9	....	....	....	....	.407	5	.408	1	.928	2	.957	19	.916	15	.979	11	.528	20	.045	7	.523	28½
ν Eri..	B2	.493	5½	.544	1	....	....	.441	2	....	....	.908	1	.959	14	.830	7	.929	5	.553	8	....	....	.516	19
γ Ori..	B2	.473	9	....	....	....	....	....	....	....	....	.863	2	.934	17	.914	3	.967	1	.473	3	....	....	.523	22
102 Her.	B2	.574	1	....	....	....	....	....	....	....	....	....	....	.948	8	.949	2	.036	1	.559	5	....	....	.493	12
ι Her.	B3	.528	2	....	....	....	....	....	....	....	....	....	....	.969	10	.888	½	....	....	.555	6	....	....	.506	16
η Lyr .	B3	.430	1½	....	....	....	....	....	....	....	....	....	....	.954	10	....	....	.925	1	.569	9	....	....	.543	12
ε Cas..	B5	.499	5½	....	....	....	....	....	....	....	....	....	....	.894	10	....	....	....	....	.538	4	....	....	.530	18
π^5 Ori..	B5	.477	10	....	....	....	....	....	....	....	....	....	....	.911	12	.996	3	.818	2	....	....	....	....	.532	23
τ Her.	B5	.482	4	....	....	....	....	....	....	....	....	....	....	.954	5	....	....	....	....	.558	3	....	....	.512	15
ζ Dra.	B5	.455	16	....	....	....	....	....	....	....	....	....	....	.974	4½	....	....	.965	3	....	....	....	....	.511	19½
67 Oph.	B5p	....	....	....	....	....	....	....	....	....	....	....	....	.886	6	....	....	....	....	....	....	....	....	.556	9
ε Del .	B5	.444	4	....	....	....	....	....	....	....	....	....	....	.881	2	.960	1	.957	1	.576	1	....	....	.513	7
β Ori..	B8p	.467	30	....	....	....	....	....	....	....	....	....	....	.949	34	....	....	.935	1	.575	4	....	....	.534	54
γ Crv .	B8	.462	2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.530	3½
Weighted mean		.471	163	.565	22	.400	10	.406	64½	.313	17	.853	69½	.946	284½	.899	188	.973	118½	.554	128	.016	10	.524	499½
No. of measures .		121		16		10		45		18		55		186		105		83		82		7		224	
Prob. error of mean..		±.004		.014		.015		.007		.012		.006		.003		.004		.005		.005		.014		±.002	
Prob. error of obs. of wt. 1.....		±.050		.067		.044		.057		.047		.053		.052		.052		.049		.048		.046		±.053	

. TABLE I—*Continued*

Star	Sp. Class	4481.229 *Mg* II		4552.608 *Si* III		4567.822 *Si* III		4574.736 *Si* III		4583.838 *Fe* II		4590.983 *O* II		4596.180 *O* II		4630.551 *N* I		4641.828 *O* II *N* II		4649.148 *O* II		4661.650 *O* II		4713.199 *He*	
		Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt.	Obs.λ	Wt	Obs.λ	Wt.	Obs.λ	Wt.
ε Ori..	B0	.168	2	.605	2½	.749	3																	.160	7
ζ Ori..	B0																								
κ Ori..	B0	.300	6	.595	15	.793	6	.759	1			.891	3	.244	1	.549	1								
ζ Per..	B1	.289	2	.585	9	.811	10½	.742	6							.664	1	.849	1			.591	2	.252	3
η Ori..	B1	.238	22½	.613	56	.804	55½	.735	44			.971	33	.205	13			.804	4	.082	7	.679	2	.141	10½
β CMa.	B1	.145	2	.638	14	.825	11	.773	4½			.946	4½	.143	2½	.549	1	.786	1	.093	4	.667	½		
ε CMa.	B1	.220	5	.582	8	.826	10	.758	6			.952	1½	.124	1½										
β Cep..	B1	.217	29	.602	64½	.844	54	.753	35			.949	37½	.181	23	.551	7	.819	2	.164	7	.683	5	.284	3
γ Peg..	B2	.217	38	.592	25	.803	25	.718	8			.935	3	.258	3	.542	3			.135	3	.623	4	.223	5
ζ Cas..	B2	.195	16	.641	12½	.821	8	.727	3½					.173	2	.501	2							.221	3
δ Cet..	B2	.202	31	.591	30	.784	27½	.774	20			.960	2			.557	2½			.130	4			.208	2½
ν Eri..	B2	.226	19½	.619	15	.818	17	.740	7			.856	3	.252	1									.238	1
γ Ori..	B2	.220	10	.639	8	.818	12	.788	2½			.946	2½							.209	2			.122	6
102 Her..	B2	.203	7	.581	10	.839	5	.740	2																
ι Her..	B3	.218	20	.576	9	.812	2																	.184	6
η Lyr..	B3	.195	18	.634	2	.750	3																		
ε Cas..	B5	.218	18																						
π⁵ Ori..	B5			.603	8½	.793	4½	.658	1					.124	2										
τ Her..	B5	.210	15																						
ζ Dra..	B5	.224	19																						
67 Oph.	B5p	.250	12	.538	4½			.770	1	.939	1													.133	4
ε Del..	B5	.235	9																						
β Ori..	B8p	.204	62	.556	1	.810	3			.814	5½													.151	7
γ Crv..	B8	.246	17							.806	1														
Weighted mean......		.217	380	.603	294½	.813	257	.747	141½	.830	7½	.951	90	.188	49	.551	17½	.811	8	.127	27	.650	13½	.175	58
No. of measures..		175		145		131		94		8		56		35		13		7		14		9		26	
Prob. error of mean...		.002		.004		.004		.004		.013		.006		.010		.011		.01		.011		.016		.008	
Prob. error of obs. of wt. 1......		.045		.058		.058		.051		.04		.056		.071		.045		.03		.055		.057		.060	

the average 0.010 A smaller, while for hydrogen, helium, enhanced magnesium, and doubly enhanced silicon as a group they are 0.003 A larger than in laboratory spectra. This moderate relative differ-

TABLE II

SUPPLEMENTARY LIST OF OBSERVED WAVE-LENGTHS
LINES WITH FEW MEASURES

Star	Sp. Class	Observed λ (I.A.)	Wt.	No. of Obs.	Normal λ	Element
β CMa	B1	4317.112	2	1	.160	*O* II
δ Cet	B2	4317.231	2	1	.160	*O* II
β CMa	B1	4319.623	½	1	.647	*O* II
δ Cet	B2	4319.622	1	1	.647	*O* II
ζ Dra	B5	4351.922	2	2	.915	*Mg* I
β Ori	B8p	4351.878	2	1	.915	*Mg* I
γ Crv	B8	4367.610	1	1	.607	....
γ Crv	B8	4395.056	1	1	.036	*Ti* II
η Ori	B1	4395.999	3	1	.950	*O* II
β Cep	B1	4395.851	1	1	.950	*O* II
γ Crv	B8	4427.266	2	2	.250	*Ti* II, *Fe* II?
δ Cet	B2	4432.605	0	1	.566	*Fe* II, *Ti* II?
γ Crv	B8	4443.842	2	2	.799	*Ti* II
β Cep	B1	4452.366	1	1	.380	*O* II
γ Crv	B8	4468.517	5½	6	.492	*Ti* II
ν Eri	B2	4469.400	1	1	.385	*Fe* II?
γ Crv	B8	4469.379	2	1	.385	*Fe* II?
η Ori	B1	4491.276	2	1	.250	*O* II
γ Crv	B8	4501.335	6	5	.269	*Ti* II
γ Crv	B8	4518.335	1	1	.334	*N* II?
β Ori	B8p	4522.634	1	1	.630	*Fe* II
γ Crv	B8	4533.951	3	2	.966	*Ti* II
δ Cet	B2	4540.658	0	1	.603	....
γ Crv	B8	4549.574	5	3	.593	*Fe* II, *Ti* II
γ Crv	B8	4552.388	1	1	.454	*Ti* II?
γ Crv	B8	4563.701	9	6	.765	*Ti* II
γ Crv	B8	4571.938	7	5	.970	*Ti* II
δ Cet	B2	4601.734	0	1	.007	....
δ Cet	B2	4607.118	1	1	.167	*N* I
γ Ori	B2	4621.384	1	1	.405	*N* I
β Cep	B1	4636.924	2	1	.922	....
ε Ori	B0	4638.700	2	1	.865	*O* II
β CMa	B1	4638.837	3	2	.865	*O* II
β Cep	B1	4638.862	1½	2	.865	*O* II
π^5 Ori	B3	4642.420	2	1	.387	....
δ Cet	B2	4642.917	0	1	.889	....
β Cep	B1	4647.482	2	1	.438	....
ζ Per	B1	4650.828	1	1	.853	*N* I, *O* II
β CMa	B1	4650.771	½	1	.853	*N* I, *O* II
β Cep	B1	4650.842	1	1	.853	*N* I, *O* II
η Ori	B1	4676.273	2	1	.246	*O* II
β CMa	B1	4676.188	½	1	.246	*O* II
β Cep	B1	4676.084	1	1	.246	*O* II
γ Ori	B2	4699.209	2	1	.210	*O* II
η Ori	B1	4705.340	1	1	.358	*O* II

ence of 0.013 A between the two groups of lines may represent either systematic errors in the normal wave-lengths, presumably mostly for the oxygen and nitrogen lines, or a real relative displacement in the stellar spectra which might be interpreted as an inward motion of *H*, *He*, *Mg* II, and *Si* III, relative to *O* II and *N* II of 0.9 km per second. In view of the serious difficulties which the enhanced spectra of oxygen and nitrogen and particularly of silicon present in the laboratory, the former alternative seems entirely possible. At any rate, it is subject to test in the laboratory and should be settled definitely before resorting to the second alternative. This will require that the normal wave-lengths, especially for enhanced oxygen and nitrogen and doubly enhanced silicon, shall receive attention in the laboratory comparable to that given to the ion spectrum.

Some of the discrepancies between the observed and the normal wave-lengths seem quite clearly attributable to inaccurate or uncertain normal, i.e., laboratory wave-lengths. Thus, unfortunately, some of the most important lines are blends of two components. For these the normal wave-lengths have quite commonly been derived by blending the components according to their relative intensities. This is true for the helium blend 4471.5, for which the adopted normal wave-length should almost certainly be increased by about 0.02 A; for the enhanced magnesium line 4481, the blended normal of which apparently should be reduced by about 0.01 A; and for the helium line 4713, for which the adopted normal is too high. It would be highly desirable that wave-lengths of laboratory blends which are to be employed as normal values for stellar spectra should be actually determined with the three-prism stellar spectrograph. Possible changes in the relative intensities of the components with changes in the experimental conditions, including different slit-widths and varying density on the spectrograms as well as effects of personality, should be included in this study, especially for the pairs in which the components are of very unequal intensity. In this way it may be possible to obtain normal wave-lengths for these blends which would have essentially fundamental value for stellar work, and in addition offer an opening wedge on the problem as to whether the components of these doublets have or have not the same relative intensities in stellar spectra as in the laboratory spectra. The

TABLE III

OBSERVED *minus* NORMAL WAVE-LENGTHS

Line	Weight	*H*γ, *He*, *Mg* II, and *Si* III	*O* II, *N*, and *N* II	Metals	Unidentified	Spectral Class and Remarks
4317.1	4	....	+.011	....	....	B1
4319.6	$1\frac{1}{2}$	....	− 25	....	....	B1
4340.4	163	+.005	....	....	....	B0–B8
4345.5	22	....	− 5	....	....	B1
4347.4	10	....	− 29	....	....	B1
4349.4	$64\frac{1}{2}$	....	− 28	....	....	B0–B2
4351.2	17	....	+ 38	....	....	B0, B1
4351.9	4	− 15	....	....	....	B5 and B8; *Mg*
4366.9	$69\frac{1}{2}$	....	− 53	....	....	B0–B2
4367.6	1	....	....	....	+.003	B8
4387.9	$284\frac{1}{2}$	+ 15	....	....	....	B0–B8
4395.0	1	....	....	+.020	....	B8; *Ti* II
4395.9	4	....	+ 12	....	....	B1
4414.8	188	....	+ 11	....	....	B0–B5
4416.9	$118\frac{1}{2}$	....	− 1	....	....	B0–B8
4427.2	2	....	....	+ 16	....	B8; *Ti* II, *Fe* II?
4432.5	0	....	....	+ 39	....	B2; *Fe* II, *Ti* II?
4437.5	128	+ 6	....	....	....	B0–B8
4443.7	2	....	....	+ 43	....	B8; *Ti* II
4447.0	10	....	− 19	....	....	B1
4452.3	1	....	− 14	....	....	B1
4468.4	$5\frac{1}{2}$	....	....	+ 25	....	B8; *Ti* II
4469.3	3	....	....	+ 1	....	B8; *Fe* II?
4471.5	$499\frac{1}{2}$	+ 19	....	....	....	B0–B8
4481.2	380	− 12	....	....	....	B0–B8
4491.2	2	....	+ 26	....	....	B1
4501.2	6	....	....	+ 66	....	B8; *Ti* II
4518.3	1	....	+ 1	....	....	B8; *N* II?
4522.6	1	....	....	+ 4	....	B8; *Fe* II
4533.9	3	....	....	− 15	....	B8; *Ti* II
4540.6	0	....	....	....	+ 55	B2
4549.5	5	....	....	− 19	....	B8; *Fe* II, *Ti* II
4552.4	1	....	....	− 66	....	B8; *Ti* II?
4552.6	$294\frac{1}{2}$	− 5	....	....	....	B0–B8
4563.7	9	....	....	− 64	....	B8; *Ti* II
4567.8	257	− 9	....	....	....	B0–B8
4571.9	7	....	....	− 32	....	B8; *Ti* II
4574.7	$141\frac{1}{2}$	+ 11	....	....	....	B0–B5
4583.8	$7\frac{1}{2}$	....	....	− 8	....	B8 and B5; *Fe* II
4590.9	90	....	− 32	....	....	B0–B2
4596.1	49	....	− 1	....	....	B0–B5
4607.1	1	....	− 49	....	....	B2
4621.4	1	....	− 21	....	....	B2
4630.5	$17\frac{1}{2}$	....	0	....	....	B0–B2
4636.9	2	....	....	....	+ 2	B1
4638.8	$6\frac{1}{2}$	....	− 64	....	....	B0 and B1
4641.8	8	....	− 17	....	....	B1
4642.3	2	....	....	....	+ 33	B3
4642.8	0	....	....	....	+ 28	B2
4647.4	2	....	....	+ 44	....	B1; *Fe*?
4649.1	27	....	− 21	....	....	B1 and B2

TABLE III—*Continued*

Line	Weight	$H\gamma$, *He*, *Mg* II, and *Si* III	*O* II, *N*, and *N* II	Metals	Unidentified	Spectral Class and Remarks
4650.8	3½	....	− 31	....	....	B1
4661.6	13½	....	0	....	....	B1 and B2
4676.2	3½	....	− 39	....	....	B1
4699.2	2	....	− 1	....	....	B2
4705.3	1	....	− 18	....	....	B1
4713.1	58	− 24	....	....	....	B0–B8
Weighted mean		+.003	−.010	−.005	+.015	
Sum of weights		2210	737	55	5	
Mean of all		....	−.000,3			
Sum of weights		....	3007			

importance of such laboratory study of the doublets can hardly be overemphasized because of the paucity of lines which have strictly fundamental value, especially in spectra in which the enhanced oxygen and nitrogen lines and the doubly enhanced silicon lines are entirely absent or are too faint for measurement. Moreover, for some stars 4471 and 4481 are either the best lines on the plates or the only ones that are suitable for accurate measurement.

The groups of lines taken separately and collectively indicate a small systematic difference between the lines in the upper and lower halves of Table III. Without attempting any corrections due to considerations pointed out above, the first six lines in the first group give a mean difference for observed *minus* normal wave-length differing by +0.0115 A from the mean for the last four lines, and the mean for the first twelve lines in the enhanced oxygen and nitrogen group differs from the corresponding mean for the last fifteen lines by +0.0125 A. The metals give a somewhat larger difference. Thus a total effect of about 0.015 A, either associated with peculiarities of certain comparison lines or due to coma, seems indicated.

Table III does not lend support to the presence of large relative radial motions between the metallic and non-metallic gases in which the respective groups of lines originate. The metallic lines rarely occur or are measured in types earlier than B5. The results for the enhanced metallic lines in Table III apply almost entirely to B8

stars, in which the enhanced oxygen and nitrogen and the doubly enhanced silicon lines are practically absent. The $H\gamma$, *He*, and *Mg* II lines constitute the elements of correlation with the metallic lines. Taking only the plates on which metallic lines were included in the measures, and excluding lines for which the normal wave-lengths are doubtful due to uncertain origin or duplicity, the non-metallic lines give a mean residual of observed *minus* normal wave-length of −0.0057 A, weight 72.5, and the metallic lines, −0.0062 A, weight 42. The indicated displacement between metallic (mostly enhanced) and non-metallic lines is therefore only 0.0005 A, corresponding to a relative motion of the respective gases of only 0.03 km/sec. This small difference is well within the limits of accuracy of the quantities involved. Hence, whatever shift the enhanced metallic lines may contain is shared by the non-metallic lines.

In Table IV are given the revised mean of the radial velocity for each star, the change from the former value and the groups of lines with their weights which form the basis for the results. The largest changes in radial velocity occurred in class B1, to which the greatest number of changes in the normal wave-lengths apply. The last column shows that each subdivision of spectra of class B contains some measures of enhanced oxygen and nitrogen and of doubly enhanced silicon. This fact is worthy of special mention in view of the prevailing belief that these lines are entirely absent in the later subdivisions of class B. The metallic lines appear only in the measures of spectra late in class B.

The foregoing results constitute an important part of the data which were employed in previous discussions[1] of the problems of the K-term, of relativity displacements and of general convection currents in the atmospheres of stars of class B. Those discussions, revealing the close relation of several astrophysical problems of a fundamental nature with observed stellar wave-lengths and radial velocities, serve to emphasize the caution which should be exercised in the choice of wave-lengths to be employed as normals. For the reduction of measures of spectra of class B, laboratory wave-lengths should be employed to the exclusion of adjusted values derived from the stellar spectra. This is for the purpose of preserving

[1] *Astrophysical Journal*, **55**, 361, 1922; **57**, 57, 1923; **63**, 277, 1926.

TABLE IV

RADIAL VELOCITIES

Star	Spectrum	R.A.	Dec.	Radial Velocity	No. of Plates	Change in Rad. Vel.	Lines Measured and Sums of Weights					
							$H\gamma$	*He*	*Mg* II	*Si* III	*O* II and *N* II	Miscellaneous
				km.								
ε Ori	B0	5^h31^m	$-1°16'$	+25.88	4	−0.9	6	30	2	6	2	......
ζ Ori	B0	5 36	− 2 00	+17.70	5	−0.1	9	24	......	......	5	......
κ Ori	B0	5 43	− 9 42	+16.63	7	−0.9	7	34	6	22	12	......
ζ Per	B1	3 48	+31 35	+20.79	5	−1.5	4	20	2	26	25	......
η Ori	B1	5 19	− 2 29	Var.	28	−2.0	11	118	22	156	236	......
β CMa	B1	6 18	−17 54	+31.0	3	−1.9	2	15	2	30	43	......
ε CMa	B1	6 55	−28 50	+26.04	3	−1.7	3	18	5	24	25	......
β Cep	B1	21 27	+70 07	Var.	21	−2.1	7	104	29	154	225	—, 4
γ Peg	B2	0 08	+14 38	+ 4.34	8	−1.0	7	67	38	58	49	......
ζ Cas	B2	0 31	+53 21	+ 1.80	4	−0.9	8	40	16	24	8	......
δ Cet	B2	2 34	− 0 06	+ 8.62	11	−1.3	9	70	31	78	54	......
ν Eri	B2	4 31	− 3 34	Var.	7	−1.2	6	42	20	39	20	*Fe* II?, 1
γ Ori	B2	5 20	+ 6 15	+16.61	7	−1.0	9	48	10	22	14	......
102 Her	B2	18 04	+20 48	−11.25	4	−0.9	1	25	7	17	3	......
ι Her	B3	17 37	+46 04	−17.38	4	−0.8	2	38	20	11	1	......
η Lyr	B3	19 10	+38 58	− 9.02	4	−0.3	2	31	18	5	1	......
ε Cas	B5	1 47	+63 11	− 6.02	4	−0.1	6	32	18	......	......	......
π^5 Ori	B5	4 49	+ 2 17	Var.	7	−0.6	10	35	......	14	7	—, 2
τ Her	B5	16 17	+46 33	−12.44	4	+0.3	4	23	15	......	......	......
ζ Dra	B5	17 08	+65 50	−14.71	4	−0.3	16	24	19	......	3	*Mg* I, 2
67 Oph	B5p	17 56	+ 2 56	− 4.26	3	−1.0	......	19	12	6	......	*Fe* II, 1
ε Del	B5	20 28	+10 58	−26.39	4	−0.2	4	10	9	......	2	......
β Ori	B8p	5 10	− 8 19	+20.86	19	0.0	30	99	62	4	1	*Fe* II, 6; *Mg* I, 2
γ Crv	B8	12 11	−16 59	− 6.37	3	+0.6	2	4	17	......	1	*Ti* II and *Fe* II, 44; —, 1

the fundamental character of the derived results. Only for special purposes and in special cases like *c* Carinae,[1] in which lines of unknown origin are involved, can such adjusted wave-lengths be of real value. The inherent nature of certain spectra in which only 4471 or 4481, or both, are measurable will necessitate the use of blended values for normal wave-lengths. These blended wave-lengths should be determined from laboratory spectra obtained with the stellar spectrograph. Until such new values become available, strongly determined stellar wave-lengths might be given preference over wave-lengths blended according to the estimated intensities of the components in laboratory or solar spectra obtained with high dispersion.

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[1] *Publications of the Lick Observatory*, **9**, 172 n., 1907.

ON THE PERIOD AND RADIAL VELOCITY OF THE CLUSTER-TYPE VARIABLE RR LYRAE[1]

By ROSCOE F. SANFORD

ABSTRACT

Changes in the period and radial-velocity variation of the cluster-type variable, RR Lyrae, have been determined from existing photometric data and two series of spectrograms, one taken at the Lick Observatory (1911 and 1912) and the other at the Mount Wilson Observatory about fifteen years later (Table I).

Period.—The following formula harmonizes all photometric and radial-velocity data:

$$\text{Max. light} = \text{J.D. } 2414856.480 \text{ G.M.T.} + 0^{d}.56681729E + 0^{d}.121\ (10^{-4}E)^2\ .$$

Radial velocity.—When the velocity-curves are defined by orbital elements it is found that the eccentricity, the angle of periastron, the time of periastron passage, and the semiamplitude of velocity variation are all larger for the Mount Wilson curve than for the Lick Observatory curve. Reasons are given for believing that the increase in the last three elements is real, while the eccentricity is probably the same in the two cases and of the order of magnitude found for the Mount Wilson curve (0.300). The values for the velocity of the system from the two series of observations are in excellent agreement.

The velocity-curve derived from the hydrogen lines on the Mount Wilson plates appears to differ from the general curve in being slightly retarded in phase and in having a flatter minimum and a sharper and higher maximum.

Cluster-type variable stars as a class are faint, and only one, the prototype, RR Lyrae,[2] is bright enough to be easily studied with a slit spectrograph of moderate dispersion. Kiess[3] discussed its variable radial velocity with the aid of thirty-three spectrograms, obtained with the 36-inch refractor at the Lick Observatory in 1911 and 1912. It is of some importance, however, to redetermine the radial-velocity variation of a cluster-type variable in order to test for possible changes in characteristics, such as appear in some Cepheids. The purpose of this paper is to study the variation of RR Lyrae as shown by twenty-nine spectrograms obtained at Mount Wilson between 1916 and 1927, and compare the result with that found by Kiess. Table I gives the data for the plates, γ indicating spectrograms made with the 60-inch and C those made with the 100-inch reflector. The dispersion is approximately 37 A per millimeter at $H\gamma$.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 351.

[2] B.D.$+42°3338$, H.D. 182989, α $19^{h}22^{m}.3$, δ $+42°\ 36'$(1900).

[3] *Lick Observatory Bulletins* **7**, 140, 1913.

An attempt to form a velocity-curve by collecting Mount Wilson observations with Kiess's period showed that the phases for maximum and minimum velocity, as reckoned from maximum light, would be quite different from those determined by him with the aid of nearly simultaneous photometric and spectroscopic observations. It was found, however, that the radial-velocity measures made at the Lick and Mount Wilson observatories could be harmonized by a mean period of $0^{d}.56684871$. Since Kiess used $0^{d}.566826$, the new value represents a substantial increase in the period.

A search was then made for data bearing on the period of RR Lyrae, and this revealed the fact that many attempts have been made to find a formula accurately representing the large number of light-maxima that have been observed. The results and those who derived them are too numerous to be mentioned individually here, but the references have been collected by Müller and Hartwig.[1] The formulae fall into three general classes: one with a constant period, a second which introduces a periodic variation in the period, and a third which includes a term representing a secular increase in the period. None of these formulae gives phases for the maximum and minimum velocities determined by the Mount Wilson data which agree with what Kiess found. One feature revealed by all formulae, when applied to the longer interval afforded by later determinations of maximum light, is that the period must in some way be increased. With this in mind it seemed best to attempt to find a formula of the above-mentioned third type to represent the times of established maxima of light.

Shütte[2] seems to have made the most recent collection of observed times of light-maxima (forty-one in all), and these I have used as a basis for deriving a formula for maximum light involving a term for the secular increase in the period. This turned out to be

$$\text{Max. light} = \text{J.D. } 2414856.490 \text{ G.M.T.} + 0^{d}.56681188E + 0^{d}.155\,(10^{-4}E)^{2}\ ,$$

with which the largest residual is 0.026 days.

It remains to be seen whether slight changes in the foregoing

[1] *Geschichte und Literatur des Lichtwechsels*, **2**, variable star numbered "1317," 1920.

[2] *Astronomische Nachrichten*, **218**, 170, 1923.

formula can be made such that it will still satisfactorily represent the light-maxima and yet give periods for the epochs of Kiess's observations and of the latest Mount Wilson observations, the mean of which will be the mean period required to superpose the two sets of measures. These conditions were found to be satisfied by

$$\text{Max. light} = \text{J.D. } 2414856.480 \text{ G.M.T.} + 0^{d}\!.56681729E + 0^{d}\!.121\ (10^{-4}E)^2 .$$

With this formula only one observed light-maximum differs as much as $0^{d}\!.020$ from the computed value. This is the photographically determined maximum by Hertzsprung for which the residual is $-0^{d}\!.037$; and only four residuals fall within the limits $0^{d}\!.020$ and $0^{d}\!.010$. The residuals for the forty-one dates of light-maximum leave no evidence for a periodic term in the period large enough to have any certainty, especially if one considers the chance of systematic error in a collection of maxima derived by a large number of observers and by various observational methods. The foregoing formula has therefore been adopted and has been used to determine the light-maxima from which the phases for the radial velocities were counted.

These phases for the Mount Wilson observations appear in Table I and are the abscissae for the velocities (ordinates) plotted as circles at the top of Figure 1, where barred circles refer to plates taken prior to 1926, and open circles to those obtained during 1926 and 1927.[1] The broken-line curve is Kiess's velocity-curve, while the full-line curve is defined by the set of orbital elements derived from the Mount Wilson observations by Russell's method.[2] The circles in the portion of Figure 1 next below are Kiess's radial velocities plotted with the phases given by the adopted formula. These observations have a slightly different arrangement from that shown in Kiess's radial-velocity diagram, because of the effect of the term in E^2 on the maxima from which phases have been reckoned, and

[1] The first three plates taken in 1926 (γ 14266, 14267, and 14268) are plotted as given in Table I, with, however, the alternative arrangement of the first two on the assumption that their numbers should be interchanged because of a mistake in marking them. A line joining the circles for these three plates is then nearly a straight line parallel to the velocity-curve. It is impossible to assert that such a change in the numbering should be made, but this seems quite probable.

[2] *Astrophysical Journal*, **40**, 282, 1914.

because of slight errors in his original plotting. These changes are minor and do not appreciably modify the velocity-curve. The plotted points are accompanied by the same two curves as in the upper part of the figure, but now with the full line for Kiess's curve, and the broken line for the Mount Wilson curve.

TABLE I

Observations of RR Lyrae—Mount Wilson

Plate	Date	G.M.T.	Phase	Velocity
				km/sec.
γ 4793......	1916 May 11	23^h28^m	0^d158	− 89.2
7081......	1918 June 25	22 56	.259	65.2
7125......	July 16	20 38	.189	76.7
C 390......	1920 Apr. 29	22 41	.265	69.3
1978......	1922 Oct. 31	14 55	.044	104.1
1979......	Oct. 31	15 25	.065	97.0
2179......	1923 Mar. 29	0 08	.479	46.2
2199......	Apr. 24	23 30	.243	73.4
γ 14266......	1926 June 20	21 36	.177	65.4*
267......	June 20	22 52	.230	78.0
268......	June 20	23 43	.265	57.4
C 3829......	June 21	17 25	.436	34.8
3830......	June 21	17 52	.455	47.4
γ 14270......	June 21	18 16	.471	49.4
C 3834......	June 21	22 15	.070	87.4
3835......	June 21	22 44	.090	99.6
3836......	June 21	23 07	.106	87.6
γ 14278......	June 22	18 11	.335	62.9
C 3845......	June 22	23 35	.560	92.4
3900......	July 23	22 55	.355	47.4
γ 14374......	July 24	16 10	.507	58.9
14376......	July 24	20 17	.111	95.6
14383......	July 25	20 47	.565	101.6
14425......	Aug. 15	21 1	.008	105.7
15029......	1927 June 12	18 22	.487	61.7
15031......	June 12	20 44	.019	105.3
15177......	Aug. 11	22 40	.012	108.6
15305......	Oct. 12	15 06	.482	58.6
15307......	Oct. 12	18 24	0.047	−104.2

* There is a possibility that the phases of these two plates should be interchanged.

To facilitate the comparison of the two velocity-curves, their description in terms of orbital elements is given in Table II. The period has already been discussed. The two values of the velocity of the system are in very good agreement. The eccentricity here derived does not differ from that found by Kiess by more than its probable error. The plot of Kiess's observations suggests, however,

that the falling branch of his velocity-curve might have been made steeper, and that his observations perhaps indicate a larger eccen-

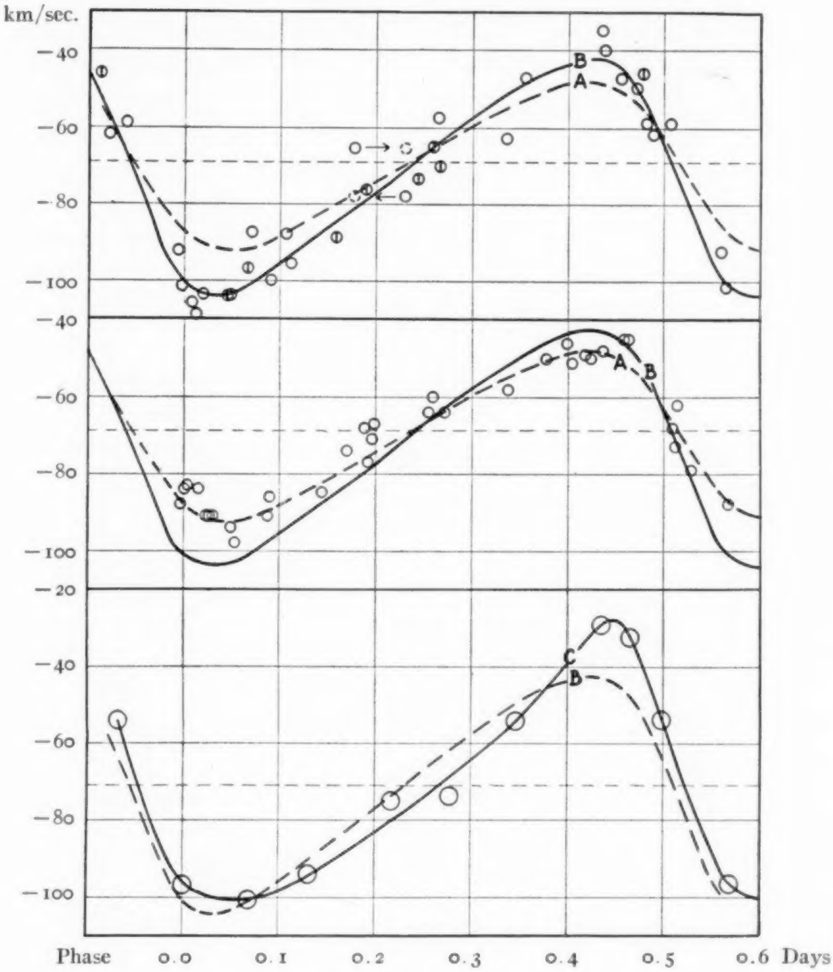

FIG. 1.—Radial-velocity curves of RR Lyrae

Top: Mount Wilson observations; barred circles indicate those prior to 1926, open circles those of 1926 and 1927. Middle: Circles represent Kiess's observations. Bottom: Circles indicate velocities derived from the $H\gamma$ line. A, B, C are, respectively, the Mount Wilson general velocity-curve, Kiess's velocity-curve, and the velocity-curve given by the $H\gamma$ line. The broken horizontal lines show the value of γ associated with each set of data.

tricity than that found. If such be the case, there remains no evidence for a change in this element, which probably is at least as large as the value here derived (0.300). There is an increase in the value of ω, the longitude of periastron, which seems larger than can be laid to the respective probable errors and which may be accepted as an indication of the direction, at least, of a real change in this element. The increase in the time of periastron passage is a natural consequence of the change in ω and the question of its reality is the same.

There remains the sixth element, the semiamplitude of velocity variation, which is found to be 8.4 km/sec. greater for the Mount

TABLE II

Elements	Kiess	Mt. Wilson
Period $= P$	....	....
Time of periastron passage as the interval after visual light-maximum $= T$	0.508 ± 0.009 day	0.524 day
Longitude of periastron $= \omega$	$96^\circ.853 \pm 7^\circ.2$	$120^\circ.6$
Eccentricity $= e$	0.271 ± 0.037	0.300
Semiamplitude of velocity variation $= K$	22.2 ± 0.8 km/sec.	30.6 km/sec.
Velocity of the system $= \gamma$	-68.7 km/sec.	-69.1 km/sec.

Wilson than for the Lick Observatory velocities. The reality of the difference can scarcely be doubted, for it is graphically demonstrated by the top and middle portions of Figure 1, where the two broken-line curves are obviously of the wrong amplitude to fit the observations with which they are plotted. The somewhat longer exposures necessary with the 36-inch refractor would tend to flatten the top and bottom of the velocity-curve derived with the aid of this instrument as compared with the curve derived from Mount Wilson spectrograms obtained with the 60-inch and 100-inch reflectors. But this is found to play only a minor rôle. The evidence here at our disposal therefore seems to point definitely to an increase in the amplitude of velocity variation. Since the Mount Wilson curve is defined in large part by observations made during 1926 and 1927, any such changes as those indicated must have occurred between 1911–1912 and 1926–1927. It is to be remarked, however, that the few early Mount Wilson plates, whose dates of observation are nearer 1912

than 1926–1927, are in good agreement with the Mount Wilson curve.

Kiess called attention to the fact that his velocity-curve indicates that minimum velocity follows maximum visual light by $0^{d}.025$. His observations, as here assembled, and the Mount Wilson observations are in substantial agreement and confirm Kiess's estimate of this lag.

The rather poor quality of the spectral lines of RR Lyrae makes it inadvisable to try to derive velocity-curves from special classes of lines. The high-level hydrogen lines, $H\delta$, $H\gamma$, and $H\beta$, which are the most conspicuous features of the spectrum, are perhaps an exception. With the velocities from these lines as measured upon the Mount Wilson plates, nine normal places have been formed, and the smooth curve drawn through their plot is given at the bottom of Figure 1. The accompanying broken-line curve is the Mount Wilson velocity-curve. These normal places define a smooth curve, to which the following comments seem to apply:

a) Maximum and minimum velocities, and indeed the entire curve for the hydrogen lines, show a slight retardation with respect to the general velocity-curve.

b) The minimum is much flatter and the maximum much sharper and higher than in the general curve, so that, as defined by orbital elements, ω and T are less, and e and K greater in the hydrogen-line curve. The value of the velocity of the center of mass derived from this curve is algebraically about 3 km/sec. less than that found by both Kiess and myself. It is certain that a flattening of the maximum of this curve would only make matters in this respect worse, and so there seems to be good evidence for a maximum at least as high as that shown in the velocity-curve for the hydrogen lines and hence for the reality of the larger value of K.

If it is correct to suppose that the hydrogen lines originate at high levels in the star, as they certainly do in the sun, these results indicate that the gases at high levels have a velocity-curve whose range exceeds the range at lower levels and whose phases are retarded with respect to those of the general curve.

Carnegie Institution of Washington
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ON THE PERIOD, VELOCITY-CURVE, AND SPECTRUM OF THE CEPHEID VARIABLE U VULPECULAE[1]

By ROSCOE F. SANFORD

ABSTRACT

Period and light-curve of U Vulpeculae.—A summary of the photometric data now available suggests that Seares's period may be slightly too short. His formula is Min. = J.D. 2414204.80 + $7^{d}98919E$ (G.M.T.).

Different light-curves show marked differences in form. Some are smooth symmetrical curves; others, asymmetrical, with irregularities on the descending branch. The phase difference $M-m$ ranges from $2^{d}.1$ to $4^{d}.0$, or one-fourth of the period.

Velocity-curve.—Four spectrograms were obtained in 1918 and twenty-six in 1926 and 1927. In order to represent the epoch of maximum velocity derived from the 1926–1927 observations, Seares's formula was changed to Min. = J.D. 2414204.38 + $7^{d}.99148E$ (G.M.T.). This also represents the light-minima but does not account satisfactorily for the velocities in 1918. This, together with systematic differences in the residuals for light-minima, suggests a variable period.

The orbital elements which define the velocity variation are: semiamplitude of velocity variation, 18.8 km/sec.; eccentricity, 0.45; angle of periastron, 58°; time of periastron passage, $0^{d}.849$ after minimum light or velocity maximum; and the velocity of the center of mass, −11.7 km/sec.

The time between velocity maximum and velocity minimum is about two days, in agreement with the asymmetrical light-curves of Yendell and Wilkens (Fig. 1). The largest deviations from the velocity-curve correspond roughly in phase with the irregularities in these light-curves. The difficulty with the period, the differences in the light-curves, and perhaps also the scattering in the velocities between phases $3^{d}.75$ and $7^{d}.20$ suggest some unusual phenomenon not yet fully revealed.

Spectrum.—Measures of relative intensity (Fig. 2) for nine enhanced lines and for $H\gamma$ show that maximum relative intensity is closely associated with velocity minimum, while minimum relative intensity is perhaps slightly in advance of velocity maximum.

The intensity of the line λ 4462 relative to λ 4454 is least at light-maximum and greatest at light-minimum, which is opposite to the change found in passing from a giant star to a dwarf. Its behavior is, however, consistent with the assumption that certain lines of low energy-level, especially those of iron, are strengthened in supergiants, such as Cepheids, because they are at a somewhat lower temperature than ordinary giants.

LIGHT-CURVE

U Vulpeculae[2] is a Cepheid variable which was discovered by Müller and Kempf[3] in 1896 and observed photometrically by various astronomers between that time and 1905, when Seares and Haynes began observations. In 1907 Seares discussed the relation of their results to those previously obtained.[4] The circumstances were pecul-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 352.

[2] B.D. +20°4200; H.D. 185059; α $19^{h}32^{m}.3$, δ +20° 6'.6 (1900).

[3] *Astronomische Nachrichten*, **146**, 37, 1897.

[4] *Laws Observatory Bulletin*, **1**, 150 (No. 10), 1907.

iar in that different observers had found differences in the interval between minimum and maximum amounting to a fourth of the period. Seares found that it was impossible to represent the maxima by a constant period, but that the minima then available were in accordance with the formula

$$\text{Min.} = \text{J.D.}2414204.80 + 7^{d}.98919E \text{ (G.M.T.)} .$$

Table I gives the data for the minima collected by Seares, the additional minima observed by Luizet[1] but not accessible to Seares,

TABLE I

U VULPECULAE—LIGHT-MINIMA

No.	E	Min. J.D. (G.M.T.)	O−C I	O−C II	Reference
1	− 85	2413524.22	−1.50	−0.88	Müller and Kempf, *A.N.* 3483
2	0	2414204.47	−0.33	+ .09	Müller and Kempf, *A.N.* 3483
3	+ 38	2414508.35	− .04	+ .28	Luizet, *A.N.* 3570
4	55	2414644.30	+ .09	+ .37	Luizet, *A.N.* 3570
5	85	2414884.00	+ .12	+ .34	Yendell, *A.J.* 513
6	126	2415211.86	+ .42	+ .55	Yendell, *A.J.* 513
7	128	2415227.39	− .03	+ .10	Luizet, *A.N.* 4181
8	136	2415291.78	+ .45	+ .56	Yendell, *A.J.* 513
9	171	2415571.10	+ .15	+ .18	Luizet, *A.N.* 4181
10	184	2415674.75	− .06	− .06	Yendell, *A.J.* 513
11	212	2415898.27	− .24	− .30	Yendell, *A.J.* 563
12	217	2415938.56	+ .11	+ .03	Luizet, *A.N.* 4181
13	231	2416049.89	− .41	− .52	Yendell, *A.J.* 563
14	265	2416321.95	+ .02	− .17	Luizet, *A.N.* 4181
15	312	2416697.60	+ .17	− .12	Luizet, *A.N.* 4181
16	355	2417041.49	+ .53	+ .14	Luizet, *A.N.* 4181
17	357	2417056.80	− .14	− .54	Wilkens, *A.N.* 4125
18	404	2417432.50	+ .07	− .44	Seares and Haynes, *Laws Bull.*, No. 10
19	404	2417432.69	+ .26	− .25	Luizet, *A.N.* 4181
20	414	2417512.50	+ .18	− .35	Seares and Haynes, *Laws Bull.*, No. 10
21	588	2418895.41	+0.96	+ .06	Jost, *A.N.* 4643
22	1133	2423258.31	+1.76	−0.42	Doberck, *Jour. des Observ.*, **8**, 10–14

a minimum derived from the observations of Jost,[2] and a recent minimum from the observations by Doberck.[3] The fourth column shows the residuals by Seares's formula. The last two suggest that his period should be lengthened slightly, if the minima are to be represented by a formula of constant period. Since the light seems to be practically constant for two or three days near minimum, the

[1] *Astronomische Nachrichten*, **175**, 86, 1907.

[2] *Ibid.*, **194**, 202, 1912.

[3] *Journal des Observateurs*, **8**, 10–14, 1925.

photometric observations do not perhaps give conclusive evidence for the necessity of a change in the period.

The striking differences in the light-curves found by different observers is illustrated in Figure 1, which shows the visual curve of Yendell,[1] the photographic curve of Wilkens,[2] and the visual curve from the observations by Seares and Haynes.[3] The asymmetrical form also appears in the observations of Wendell as reported by Pickering,[4] in those of Doberck,[5] and in Luizet's final curve.[6] The curves of Müller and Kempf[4] and the first curve by Luizet,[7] on the other hand, display the symmetry found by Seares. The difference in form is still to be explained, although the close commensurability of the period with one day may account for part of it. It is perhaps also noteworthy that the curves of Yendell and Wilkens and the final curve by Luizet show an inflection in the decline to minimum light, a feature not appearing in the curves of the other observers. The amplitude of the photographic light-curve by Wilkens is about 1.6 times that of the visual curves, as is usually the case with Cepheids.

RADIAL VELOCITY

Four slit spectrograms of U Vulpeculae were obtained at Mount Wilson in 1918 and twenty-six more in 1926 and 1927, equally divided between the two years and well distributed over the light-curve. Table II gives the data for these spectrograms, all of which have a dispersion of about 37 A per millimeter at $H\gamma$. As usual, the prefix γ indicates plates made with the 60-inch reflector and C those made with the 100-inch reflector. For the most part, the measures have been reduced with the aid of a table of wave-lengths based upon spectrograms of T Monocerotis at median magnitude, a procedure which has proved quite satisfactory.

In attempting to form a velocity-curve the radial velocities were first assembled according to the phases furnished by Seares's formula. The result was a consistent grouping, except that the velocity maximum occurred more than two days after the predicted mini-

[1] *Astronomical Journal*, **22**, 75, 1901.

[2] *Astronomische Nachrichten*, **172**, 331, 1906.

[3] *Loc. cit.*

[4] *Astronomische Nachrichten*, **149**, 171, 1899.

[5] *Loc. cit.*

[6] *Astronomische Nachrichten*, **175**, 87, 1907.

[7] *Ibid.*, **149**, 316, 1899.

mum of light. This would mean that velocity maximum and maximum light are nearly coincident, whereas it is certain that this phase

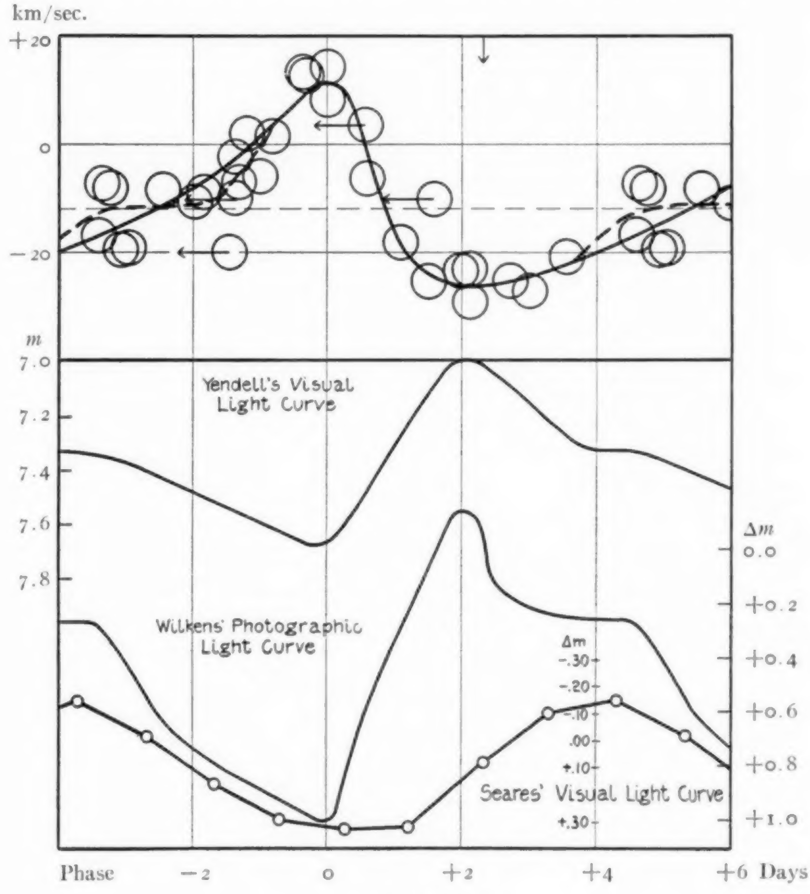

FIG. 1.—U Vulpeculae

Top: Circles represent individual velocities; full line is curve from elliptic elements, with possible modification as shown by broken-line portion; broken horizontal line shows the γ velocity. Bottom: Three light-curves. Ordinates for Yendell's are magnitudes as indicated on the left; those for Wilkens' are Δm, as given in the margin on the right.

for velocity should be correlated with minimum light. This is shown by the fact that, under identical observing conditions, plates which gave maximum velocity required the longest exposure and showed

the greatest effective wave-length, while those which gave minimum velocity received the shortest exposures and showed the shortest effective wave-length. Moreover, the association of velocity maximum with light-minimum, and vice versa, is characteristic of all Cepheids for which studies based on slit spectrograms have been made. The problem was therefore to adjust Seares's formula so that it would represent the light-minima determined photometrically and, at the same time, the light-minimum corresponding to the velocity maximum as given by the 1926–1927 spectrograms.

A few trials led finally to the formula

$$\text{Min.} = \text{J.D.} 2414204.38 + 7^{\text{d}}.99148E \text{ (G.M.T.)} .$$

The corresponding residuals are in the fifth column of Table I. Although some of the residuals are slightly increased, the representation, on the whole, is quite satisfactory, with the agreement for Jost's and Doberck's minima much improved. Further, the grouping of the radial velocities in Table II according to the phases reckoned from minimum by the new formula shows that the coincidence of maximum velocity and minimum light has been attained (Fig. 1). Moreover, the interval which separates the velocity maximum from the next following velocity minimum accords well with the values of $M-m$ for the two light-curves of Yendell and Wilkens. The asymmetrical light-curves are therefore in better agreement with the velocities than the symmetrical curves of Müller and Kempf, Luizet, and Seares.

Orbital elements derived by Russell's[1] method from the radial velocities are as follows:

P = Period	$7^{\text{d}}.99148$
e = Eccentricity	0.45
K = Semiamplitude of velocity variation	18.8 km/sec.
T = Time of periastron passage (interval after minimum light)	$0^{\text{d}}.849$
ω = Angle of periastron	58°
γ = Velocity of the system	−11.7 km/sec.

These elements define the curve shown at the top of Figure 1, in which the broken horizontal line represents the velocity of the sys-

[1] *Astrophysical Journal*, **40**, 282, 1914.

tem. The four circles with arrows attached represent velocities derived from the spectrograms of 1918. The length of these arrows indicates the effect on the velocities produced by using Seares's period. The agreement relative to the 1926–1927 observations would be improved; but, as already pointed out, such a procedure would shift

TABLE II

OBSERVATIONS OF U VULPECULAE

Plate No.	Date	G.M.T.	Phase	Velocity
				km/sec.
γ 6906......	1918 May 18	23^h29^m	0^d552	+ 3.6
6941......	May 27	22 55	1.609	−10.0
7080......	June 25	21 18	6.567	−20.1
7147......	July 19	22 10	6.631	− 9.9
14256......	1926 June 18	23 23	4.764	− 8.2
C 3894......	July 22	23 23	6.791	+ 2.0
3898......	July 23	20 18	7.669	+12.6
γ 14375......	July 24	18 46	0.614	− 6.5
14379......	July 25	16 10	1.506	−25.4
14389......	July 26	21 06	2.711	−25.4
C 3923......	July 28	19 38	4.650	− 7.2
γ 14424......	Aug. 15	19 24	6.658	− 7.4
14428......	Aug. 16	19 00	7.642	+13.2
C 4001......	Sept. 15	16 33	5.573	− 8.2
4007......	Sept. 16	17 20	6.605	− 2.4
γ 14628......	Oct. 23	15 12	3.559	−20.8
14635......	Oct. 24	15 54	4.589	−16.6
15038......	1927 June 13	21 14	5.058	−19.3
15085......	July 8	23 06	6.162	− 8.9
C 4341......	July 9	23 20	7.171	+ 1.3
4343......	July 10	18 53	7.986	+ 8.3
γ 15104......	July 11	21 16	1.094	−18.1
15109......	July 12	21 41	2.112	−29.3
15175......	Aug. 11	18 39	0.018	+14.4
15185......	Aug. 13	21 11	2.124	−23.1
C 4426......	Sept. 11	17 02	6.977	− 5.8
4450......	Oct. 8	16 13	1.977	−23.4
γ 15282......	Oct. 9	17 08	3.015	−27.3
C 4466......	Oct. 11	15 04	4.929	−19.5
γ 15306......	Oct. 12	17 00	6.009	−10.9

the velocity maximum to the phase indicated by the arrow at the top of the figure. Reasons have already been given for not accepting such a phase for velocity maximum.

The mean period which reconciles all light-minima and the velocity maximum derived from the 1926–1927 observations does not, therefore, give the best representation of the 1918 observations. The systematic character of the residuals in Table I gives some indication

of change in the period, and the failure of the 1918 observations to agree with those of 1926–1927 may likewise find explanation in a slow variation (perhaps periodic) in the period. The case for variability of period may also be stated by saying that the best period derivable from the radial velocities alone is too short to reconcile all the minima observed or inferred.

The phase interval on the rising branch of the velocity-curve in which the radial velocities have their greatest scattering corresponds to those portions of the light-curves of Yendell and Wilkens which show inflections or pauses in the decline to minimum light. In fact, the velocity-curve might well be modified as indicated by the broken-line curve between phase limits $3^{d}.75$ and $7^{d}.20$, which would emphasize the correspondence with the asymmetrical light-curves.

At the same time, it must be recalled that independent measures by three different observers have all led to highly symmetrical light-curves which appear to be free from irregularities. The anomalies affecting the period and the form of the light-curve suggest some underlying phenomenon which is still to be explained. This may account for the spread in the values of radial velocity between $3^{d}.75$ and $7^{d}.20$ after minimum light, for these are scattered over many different cycles.

THE SPECTRUM

The Mount Wilson spectral classification of U Vulpeculae is cG4 at median magnitude, with a total variation of 0.4 of a spectral division. The spectrum, as is usual for Cepheids, is characterized by great strength of the enhanced lines. A casual inspection is sufficient to show the intimate connection between the phase of maximum intensity for these lines and of minimum velocity or maximum light.

To obtain more definite information on the change of line-intensity with phase, I have compared ten prominent sensitive lines in the photographic region of the spectrum with a nearby insensitive line of the neutral atom. The lines used and their probable sources are given in Table III. The variable lines include $H\gamma$ and nine prominent enhanced lines, of which seven belong to titanium and one each to iron and magnesium. The relative intensity of λ 4462 and λ 4454, which has been useful for absolute-magnitude determinations within some spectral classes, was also measured.

A Hartmann microphotometer, as modified by Pettit to give continuous self-registry, was used to record photographic density curves of the lines and the adjoining continuous spectrum on all but the three weakest plates. Although the density probably depends to some extent on line width, as well as on the absorption at the core of the line, it was assumed that the maximum deflection registered for a line is a direct measure of its intensity. The intensity of a sensitive line relative to its standard was thus expressed by the ratio of the corresponding deflections. The results from the various plates

TABLE III

Variable Line	Source	Standard Line	Source
λ 4233	*Fe*+	4236	*Fe*
4321	*Ti*+	4325	*Fe*
4330	*Ti*+	4325	*Fe*
4334	*Ti*+	4325	*Fe*
4337	*Ti*+	4325	*Fe*
4340	$H\gamma$	4325	*Fe*
4344	*Ti*+	4325	*Fe*
4352	*Fe*+	4325	*Fe*
4468	*Ti*+	4454	Blend
4481	*Mg*+	4454	Blend

were combined into five groups having the mean phases 0.0, 1.1., 2.4, 4.9, and 6.8 days, which furnish a mean (the first) for light-minimum, another (the third) very close to light-maximum, and three others representative of intermediate stages. The corresponding means of the ratio of line intensity for each sensitive line and its standard were then plotted against these phases. The plots and the curves drawn through them are shown in Figure 2. The scale of ordinates is in some cases only one-half that used in the others, a circumstance which should be considered in judging the sensitiveness of a line. The phases of maximum and minimum of light are marked by vertical lines.

When the uncertainties involved are considered, the evidence seems to favor the coincidence of maximum relative intensity and light-maximum, while minimum relative intensity appears, on the whole, to precede slightly the minimum of light. The result is similar to that found for a few lines in the spectrum of T Monocerotis,[1]

[1] *Mt. Wilson Contr.*, No. 340; *Astrophysical Journal*, **66**, 170, 1927.

although minimum relative intensity for U Vulpeculae appears to precede minimum light less, relatively, than it does in T Monocerotis.

The relative intensity of λ 4462 and λ 4454 is a useful criterion for distinguishing giant stars from dwarfs within a certain spectral

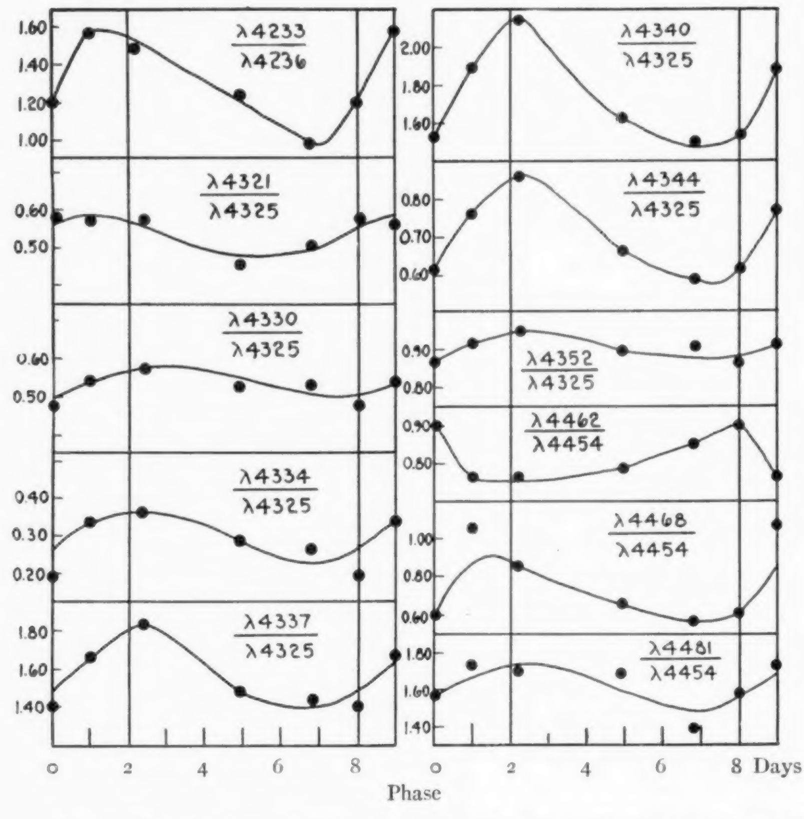

FIG. 2.—Relative intensity-curves for pairs of lines in the spectrum of U Vulpeculae.

range which includes U Vulpeculae. It is well known from recent work at Mount Wilson, however, that the low-temperature lines of different elements, especially those of iron, are very prominent in the spectra of supergiant stars such as Cepheid variables. The line λ 4258 has been used extensively as a criterion for absolute magni-

tude among supergiants, and λ 4462 is a similar line, arising from nearly the lowest energy-level. Hence we should expect it to be strong in supergiants, and such is the case in U Vulpeculae. It seems probable that the strength of these low-temperature lines in supergiants is to be ascribed to the lower temperature of these stars as compared with ordinary giants. If such is the case, it is to be expected that any lowering of the temperature would still further strengthen such lines. For a Cepheid variable the temperature changes throughout the cycle of variation. The curve for the intensity of λ 4462 relative to λ 4454 (Fig. 2) shows that it has its greatest strength at minimum light (minimum temperature) and its least at maximum light (maximum temperature), with a progressive change throughout the light-cycle. The results are therefore consistent with the hypothesis that the strengthening of λ 4462 in passing from an ordinary giant to a supergiant is the effect of a decrease in temperature. Moreover, the result for this line pair is further evidence that it cannot serve as a criterion of absolute magnitude for supergiants in the sense that it serves for ordinary giants and dwarfs, where, as has been found by the Mount Wilson observers, λ 4462 has relatively its greatest strength for the most luminous stars.

MOUNT WILSON OBSERVATORY
CARNEGIE INSTITUTION OF WASHINGTON
January 1928

ORBIT OF THE SPECTROSCOPIC BINARY 4 β TRIANGULI

By O. STRUVE and A. POGO

ABSTRACT

The *orbital elements* of this spectroscopic binary were derived from 41 spectrograms obtained with the Bruce spectrograph of the Yerkes Observatory. A least-squares solution was made using only 22 spectrograms taken in 1927–1928. $\gamma = +10.4$ km/sec.; $P = 31^{d}4009$; $e = 0.456$; $K = 26.1$ km/sec.; $\omega = 293^{\circ}9$; $T = 1927$ Nov. 6.660 U.T.; $a \sin i = 9{,}930{,}000$ km.; the mass function is 0.0397. The measurements refer to the *brighter component* of the system. The presence of the spectrum of the *fainter component* is suspected at maximum relative velocity.

Variations in the radial velocity of 4 β Trianguli were first suspected by Professor Edwin B. Frost[1] and later established by Professor S. A. Mitchell[2] from measures of the first twelve Yerkes one-prism spectrograms which seemed to indicate a period of about one month. The position of this star, for 1900, is $\alpha = 2^{h}3^{m}6$; $\delta = +34^{\circ}31'$. According to the *Henry Draper Catalogue*, its spectral type is A5 and its photometric magnitude is 3.08. The spectrum contains many metallic lines of fair quality. Most of them are rather broad and have diffuse edges, but their central intensity is sufficiently strong to make them suitable for measurement. Only one spectral component could definitely be seen. However, as the lines are strongest when the measured velocity is close to that of the system, we believe that the fainter component is strong enough to leave an impression on the photographic plate. In a few instances there appear on plates taken near maximum velocity traces of real duplicities in the lines. But these were not used as the separation was never quite complete. The published measures refer to the stronger component only. On good plates from fifteen to twenty stellar lines were usually measured.

Table I contains the radial velocities used in this paper. Professor Mitchell's measures are reprinted for the sake of completeness.[3]

[1] *Science*, **28**, 853, 1908. [2] *Astrophysical Journal*, **30**, 240, 1909.

[3] There is a misprint in the original list of velocities for 4 β Trianguli, in *Astrophysical Journal*, **30**, 240, 1909. On September 18, 1908, the velocity was measured as +9.0 km/sec., and not as −9.0 km/sec., as stated.

TABLE I

RADIAL VELOCITIES OF 4 β TRIANGULI

Date	G.M.T.	Early Observations			
		Observer	Qual.	Velocity	
1906 Dec.	28.507	B	f	+14.7	....
1907 Sep.	23.855	L	g	− 6.8	....
Oct.	7.854	L	f	+23.3	....
Oct.	11.700	B	g	+ 1.3	....
Oct.	18.712	F	g	− 9.7	....
Oct.	22.685	F	g	− 7.3	....
1908 Sep.	7.866	L	f	+54.4	....
Sep.	8.884	B	g	+52.5	....
Sep.	18.788	B	g	+ 9.0	....
Oct.	5.670	L	f	+ 2.1	....
Oct.	12.707	L	g	+33.5	....
Nov.	8.698	F	g	+30.6	....
Nov.	16.578	L, S	g	+13.9	....
Nov.	21.621	F, S	g	− 1.6	....
1909 Jan.	3.503	F, B	g	−19.6	....
Aug.	30.860	B, S	g	− 0.6	....
Sep.	10.784	L, S	p	− 5.6	....
Sep.	20.728	L, S	f	+28.3	....
1913 Jan.	29.622	M, S	g	+24.3	....
Date	U.T.	New Observations			
		Observer	Qual.	Velocity	(O.−C.)
1927 Oct.	11.344	B, S	g	+43.7	+10.3
Oct.	16.362	σ, S	g	+17.8	+ 0.4
Oct.	17.203	σ, S	g	+12.7	− 2.6
Oct.	17.467	B, S	p	+14.6	0.0
Oct.	21.056	B, P	g	+ 7.0	+ 0.5
Oct.	22.044	σ, P	g	+ 4.0	− 0.5
Oct.	25.034	σ, P	g	+ 0.8	+ 2.0
Oct.	26.040	B, P	g	− 2.8	+ 0.2
Oct.	27.116	P, S	g	− 2.3	+ 2.5
Oct.	31.269	P, S	g	−11.4	− 1.0
Nov.	4.024	B	g	− 4.2	+ 1.0
Nov.	5.117	P, S	g	+ 4.1	+ 0.2
Nov.	19.260	B, S	g	+10.3	− 1.0
Dec.	8.969	B, P, H	f	+33.7	− 2.7
Dec.	17.069	B, P, S	f	+10.8	− 9.6
Dec.	20.207	H, S	g	+10.4	− 1.6
Dec.	21.138	P, S	g	+ 8.5	− 1.7
Dec.	25.030	σ	f	− 3.4	− 5.6
1928 Jan.	12.115	B, S	g	+38.3	− 0.1
Jan.	22.085	B, P, S	g	+ 9.6	+ 0.4
Jan.	26.052	B	g	+ 5.7	+ 4.7
Feb.	5.988	B, P	g	−12.5	− 6.0

The names of the observers and the quality of the plates are designated as follows: B=S. B. Barrett; F=E. B. Frost; H=C. Hujer; L=O. J. Lee; M=G. S. Monk; P=A. Pogo; S=F. R. Sullivan; σ=O. Struve; g=good; f=fair; p=poor. The decimal fractions of the day are expressed in Greenwich Mean Time for all observations prior to January 1, 1925, and in Universal Time for all later observations.

The period was determined by trial from the observations of 1927–1928, and was then adjusted to represent the earlier observations. A preliminary velocity-curve was drawn through both old and new observations, and the elements were derived by means of Professor Laves's hodographic circle, centered on the mean axis, as proposed by Pogo.[1]

A least-squares solution for five unknowns was then made, using only the observations of 1927–1928. The period was assumed as

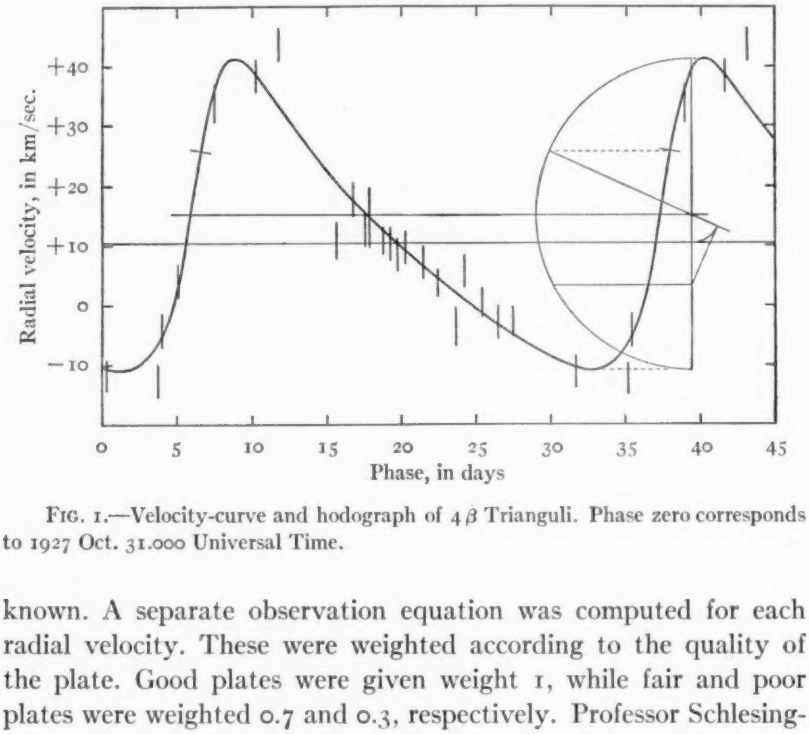

FIG. 1.—Velocity-curve and hodograph of 4 β Trianguli. Phase zero corresponds to 1927 Oct. 31.000 Universal Time.

known. A separate observation equation was computed for each radial velocity. These were weighted according to the quality of the plate. Good plates were given weight 1, while fair and poor plates were weighted 0.7 and 0.3, respectively. Professor Schlesinger's method was closely followed in the computation of the coefficients of the normal equations. The improvement of the representation is very considerable, $[p\Delta^2]$ being reduced from 649.06 to 282.11. This is due chiefly to the fact that for the preliminary orbit we used all observations, some of which gave slightly discordant values. The

[1] *Astrophysical Journal*, **67**, 262, 1928.

solution, on the other hand, was made wholly on the basis of the new observations. The probable error of one observation of unit weight is ± 2.75 km/sec. The elements are collected in Table II.

The final velocity-curve is shown in the figure. Zero phase corresponds to 1927 October 31.000 U.T. The observations of 1927–1928 are indicated by vertical lines, their lengths being equal to twice their respective probable errors.

The final values of $a \sin i$ and of the mass function were found with the aid of the hodographic construction. The essential part of the hodographic circle corresponding to the final values of K, ω, and

TABLE II

ORBITAL ELEMENTS OF 4 β TRIANGULI

Element	Designation	Preliminary	Final	Probable Error
Velocity of system.	γ	+9.0 km/sec.	+10.4 km/sec.	....
Period.	P	31ᵈ4009	31ᵈ4009	....
Time of periastron	T	1927 Nov. 8.7 U.T.	1927 Nov. 6.660 U.T.	±0ᵈ507
Longitude of periastron.	ω	306°6	293°9	±6°2
Eccentricity.	e	0.446	0.456	±0.042
Half-amplitude.	K	29.0 km/sec.	26.1 km/sec.	±1.3 km/sec.
Major semiaxis.	$a \sin i$	11,200,000 km	9,930,000 km	....
Mass function.	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.0574	0.0397	....

e is shown on the right of the figure. Its center is on the Schwarzschild S-axis, its radius is equal to the half-amplitude K. The longitude of periastron, as read off between the inclined nodal diameter and the vertical apsidal diameter, is 294°. The ratio of the distance between the origin and the center of the hodographic circle, to its radius K, gives 0.455 as the value of the eccentricity of the orbit. The horizontal half-chord through the origin of the hodograph represents $K\sqrt{1-e^2}$, the velocity across the minor axis of the inclined orbit, and corresponds to 23.0 km/sec. on the scale of radial velocities of the curve. The length, in kilometers, of the major semiaxis of the inclined orbit is, therefore, given by the product $13,750 \times 31.4009 \times 23$, and the value of the mass function by the product 1.04×10^{-7}

$\times 31.4009 \times 23^3$. It takes less time to arrive at these results by constructing the hodograph of the final velocity-curve than by using the formulae for logarithmic computation of a sin i and of the mass function.

We wish to express our thanks to Professor S. B. Barrett for many of the plates used in this paper.

YERKES OBSERVATORY
UNIVERSITY OF CHICAGO
February 13, 1928

H.D. 163181, A SPECTROSCOPIC BINARY[1]

By MILTON L. HUMASON and SETH B. NICHOLSON

ABSTRACT

Bright hydrogen lines were discovered in the spectrum of H.D. 163181 during the systematic search for Be stars carried on at Mount Wilson by means of objective-prism photographs on red-sensitive plates.

Slit spectrograms indicated that it is a spectroscopic binary with a large range of velocity and that the bright lines are variable in intensity. The period is 12.0040 days and the velocity range nearly 400 km/sec. The eccentricity is small, and the mass function, $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$, equals 8.84☉. The system is therefore one of the most massive known.

The southern star H.D. 163181, C.D. $-32^\circ 13517$, R.A. $17^h 49^m.7$, Dec. $-32^\circ 27'$ (1900), mag. 6.6, is classified in the *Draper Catalogue* as Oe5, with the remark, "The lines are so indistinct that the spectrum appears at first glance to be continuous."

Observations of the star were begun at Mount Wilson in 1920 when $H\alpha$ was found to be bright on a 10-inch objective-prism plate taken in the survey for the discovery of Be stars by Merrill, Humason, and Burwell.[2] In accordance with their program, observations were begun with a slit spectrograph and the star was classified as B1e. As in other Be stars, $H\alpha$ is much the strongest of the bright hydrogen lines. $H\gamma$ is the last of the series to appear bright. The presence of emission lines in the spectrum and the fact that the absorption lines are weak at some phases would probably explain the remark in the *Draper Catalogue* that the spectrum appears continuous.

The first slit spectrogram was obtained in 1921, and an additional one in each of the following years, 1922, 1923, and 1924. Measurements of the radial velocity from these four plates showed that the star is a spectroscopic binary with a large range in velocity. In 1925 a series of twenty-seven plates was obtained, which, with the early plates, gave sufficient material for the computation of the preliminary orbit. The resulting period of twelve days and a veloc-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 353.

[2] *Publications of the Astronomical Society of the Pacific*, **34**, 351, 1922.

ity range of almost 400 km/sec. showed that the system is very massive and similar to B.D.+6°1309, the orbit and mass of which were determined by J. S. Plaskett.[1] In 1926 and 1927 ten additional

TABLE I

OBSERVATIONS OF H.D 163181

Plate No.	Date	J. D. 2420000+	Phase in Days	Velocity	O−C	Normal Place
		G.M.T.		km/sec.	km/sec	
C 1196	1921 Aug. 12	2914.648	4.287	−179.1	− 5.2	3
1782	1922 July 11	3247.754	1.281	+ 93.9	−14.9	13
2345	1923 July 4	3605.759	11.168	+132.2	− 7.6	10
2850	1924 June 23	3960.833	6.124	−229.8	− 9.2	5
3226	1925 Mar. 18	4228.025	9.228	− 26.9	− 4.9	8
3231	Apr. 3	4244.020	1.215	+104.1	− 9.8	13
3244	Apr. 10	4251.025	8.220	−114.4	− 0.5	7
3247	Apr. 12	4253.993	11.180	+150.9	+10.5	10
3253	May 1	4272.000	5.187	−219.5	− 4.9	4
3258	May 1	4272.990	6.177	−240.6	−20.7	5
3263	May 3	4274.000	7.187	−188.1	− 3.2	6
3270	May 3	4274.959	8.146	−116.8	+ 3.1	7
3278	May 5	4276.002	9.194	− 28.1	− 2.3	8
3283	May 5	4276.972	10.159	+ 62.8	− 3.5	9
3288	May 6	4277.973	11.160	+127.3	−12.0	10
3295*	May 8	4279.991	1.174	+115.8	− 1.1	13
3302	May 29	4300.904	10.083	+ 62.2	+ 2.7	9
3307	May 30	4301.956	11.135	+135.1	− 3.0	10
3314	June 5	4307.830	5.005	−218.1	− 9.0	4
3319*	June 6	4308.907	6.082	−199.9	+21.2	5
3323	June 7	4309.898	7.073	−173.9	+16.8	6
3329	June 11	4313.798	10.973	+142.3	+13.5	10
3333	June 12	4314.835	0.006	+158.2	− 3.9	11
3340	June 13	4315.848	1.019	+125.9	− 1.8	12
3342	June 14	4316.876	2.047	+ 57.1	+23.2	1
3346	June 15	4317.859	3.030	− 91.0	−24.4	2
3351	June 16	4318.857	4.028	−177.1	−21.0	3
3365	July 2	4334.751	7.918	−139.4	− 1.7	7
3381	July 6	4338.792	11.959	+168.5	+ 6.5	11
3390	July 10	4342.659	3.822	−152.2	−12.1	3
3398	July 12	4344.705	5.868	−208.5	+14.1	5
3787	1926 May 2	4638.959	0.022	+166.2	+ 4.1	11
3825	June 20	4687.859	0.906	+130.3	− 4.1	12
3832	June 21	4688.855	1.902	+ 74.0	+21.7	1
3874	July 1	4698.854	11.901	+138.1	−23.6	11
3991	Aug. 24	4752.670	5.697	−217.6	+ 4.8	5
3997*	Aug. 25	4753.661	6.688	−206.7	+ 0.2	5
4236	1927 Apr. 18	4989.014	1.961	+ 45.4	− 0.8	1
4280	May 22	5023.908	0.843	+151.1	+12.8	12
4315	June 17	5049.853	2.780	− 8.4	+32.2	2
4416	Sept. 9	5133.652	2.551	+ 2.8	+19.1	2

* Ten-inch camera.

[1] *Monthly Notices, Royal Astronomical Society*, **82**, 447, 1922.

plates were obtained. All the observations were then divided into thirteen normal places and corrections to the elements were derived by the method of least squares.

On account of the southern declination of the star, all plates have been made with the 100-inch Hooker telescope. The regular Cassegrain spectrograph has been used with one prism and the 18-inch camera for all but three of the plates. For these three plates a 10-inch camera was substituted. The observations are given in Table I. The decimal of a day in the third column is reckoned from Greenwich mean noon, and the phase, in the fourth column, in days, is from the time of maximum velocity.

TABLE II

ELEMENTS OF H.D. 163181

	Preliminary Elements	Final Elements
P	12.0040 days	12.0040 days
e	0.075	0.065
ω	23°0	23°2
K	200.0 km/sec.	192.4 km/sec.
T	J.D. 2424279.453 G.M.T.	2424279.497 G.M.T.
γ	−44.0 km/sec.	−41.8 km/sec.
$a \sin i$	....	31,700,000 km
$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	....	8.84 ⊙

The ever changing character and intensity of the hydrogen emission seem to affect the measured position of the hydrogen absorption lines to some extent, apparently producing rather large negative residuals at most phases. The velocities given in Table I therefore do not include those obtained from the hydrogen lines.

The preliminary and final elements are given in Table II. The velocity of the center of mass, 41.8 km/sec., is rather large for the ordinary B-type star, for which the average velocity is of the order of 9.0 km/sec. The velocity-curve calculated from these elements and the individual observations are plotted in Figure 1.

Although plates at maximum and minimum velocity were examined carefully, no lines in the secondary spectrum were observed. One widened spectrum taken on a fine-grained plate near maximum velocity also failed to show lines belonging to the secondary spectrum. With only one spectrum available, the values of $m \sin^3 i$ can-

not be obtained as they can for Plaskett's star B.D.+6°1309. The mass function $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ for B.D.+6°1309, as derived for the primary, is 13.16, as compared with 8.84 for H.D. 163181. Assuming $m=m_1$, we find $m \sin^3 i = 35.4$ ☉ for H.D. 163181, and, with the same as-

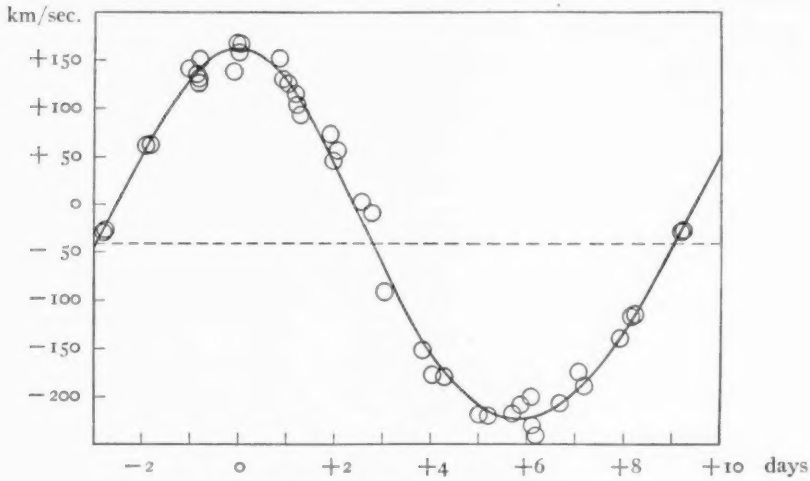

FIG. 1.—Velocity-curve for H.D. 163181

sumption, 52.6 ☉ for B.D.+6°1309. The values for the individual masses of B.D.+6°1309 are

$$m_1 \sin^3 i = 75.6 \odot$$
$$m_2 \sin^3 i = 63.3 \odot$$

The mass of H.D. 163181 is therefore probably somewhat less than that of B.D.+6°1309, although of about the same order of magnitude.

On account of the southern declination of H.D. 163181, only one spectrogram was obtained in a night, and the possibility of a period of about one day must be given careful consideration. The observations are fairly well represented with a period of 1.08759 days. Eight of them, however, give residuals greater than 30 km/sec., and the mean residual without regard to sign is 19 km/sec. The period 0.92081 days also gives a fair representation, with ten residuals greater than 30 km/sec. and a mean residual of 23 km/sec. With the adopted period 12.004 days, only one residual is greater than

30 km/sec., and the mean residual is 10 km/sec. Although the long period is therefore probably the true one, attempts will be made to obtain two plates in a night, so that the question of a short period may be still more definitely settled.

The southern declination of H.D. 163181 means a loss of blue light when observed from Mount Wilson, and it has been rather difficult to obtain the proper amount of exposure to the violet of λ 4100. This difficulty may also be partly accounted for by the fact that the star seems to be abnormally red. The *Draper Catalogue* gives the photometric magnitude as 6.62 and the photographic magnitude as 7.5. The resulting color-index is +0.9, approximately that of a star of type K0, making it over a magnitude redder than stars of its class, B1, for which the normal color-index is −0.22.

TABLE III

LINES IN H.D. 163181

λ	Element	Int.	λ	Element	Int.	λ	Element	Int.
3933	*Ca*	......	4241	*N*	1	4601	*N*	1
3964	*He*	......	4253	*S* ?	0	4607	*N*	1
3968	*Ca*	......	4340	$H\gamma$	......	4621	*N*	1
3970	$H\epsilon$	......	4387	*He*	5	4641	*O*	2
4026	*He*	1	4447	*N*	0	4685	*O*	......
4088	*Si*	3	4471	*He*	8	4713	*He*	3
4097	N^+	1	4481	*Mg*	1	4861	$H\beta$	......
4101	$H\delta$	......	4552	*Si*	5	4921	*He*	3
4116	*Si*	2	4567	*Si*	3	5015	*He*	......
4120	*He*	1	4574	*Si*	1			

The wave-lengths, intensities, and identifications of the absorption lines in H.D. 163181 are given in Table III. The intensities were estimated near phase 9.5 days, when the absorption lines have their maximum intensity.

A number of the lines in Table III have been measured on a few of the better plates only. The intensity of the lines varies with the phase, although for the weaker lines the apparent variation is probably affected by the exposure and the quality of the individual plates.

The hydrogen lines exhibit more conspicuous variation in structure than do the other lines, and are therefore ill adapted to velocity measurements.

Weak emission can usually be seen on the long wave-length edge of the dark helium lines, especially λ 4471. About two days before maximum velocity, however, the dark portions of the lines are relatively strong and the emission is very weak or absent.

The presence of emission near the helium lines has been observed in at least one other star of large mass, Plaskett's star B.D.+6°1309;[1] and this same condition has been mentioned in connection with 27 Canis Majoris[2] by Struve, who calls attention to observations by Merrill, which indicate that the helium lines are possibly accompanied by bright edges.[3] Struve states, however, that the helium lines "appear perfectly dark on all of our plates."

When the displacement of the absorption lines in H.D. 163181 is to the red, especially near maximum velocity, the emission is stronger on the violet edge; and when the displacement of the absorption lines is to the violet, the emission on the red edge is the stronger.

The absorption spectrum in general reaches a maximum at phase 9.5 days. A single plate taken at phase 3.0 days shows the emission very much stronger than at any other phase. The strengthening of the emission at this phase must occur in a very short interval, as a plate taken 0.3 day earlier and one 0.8 day later show the emission to be about normal. This is also the only phase where the emission at $H\beta$ is single and appears almost central on a wide absorption line. At all other phases the emission appears at one or both edges of the dark line.

The calcium lines H and K are present, but only two of the plates have sufficient density to show them. These two plates are decidedly under-exposed for this region, and the calcium lines are very weak and narrow. They are apparently stationary, but the measures have little weight.

Mount Wilson Observatory
Carnegie Institution of Washington
January 1928

[1] *Monthly Notices, Royal Astronomical Society*, **82**, 448, 1922.

[2] *Astrophysical Journal*, **65**, 284, 1927.

[3] *Lick Observatory Bulletin*, **7**, 170 (No. 237), 1913.

REVIEWS

Spectroscopy. By E. C. C. BALY. 3d ed. in 4 vols. New York and London: Longmans, Green & Co., Vol. I, 1924, pp. xi+298, Figs. 138, \$4.75; Vol. II, 1927, pp. viii+398, Plates 3, Figs. 95, \$6.00; Vol. III, 1927, pp. viii+532, Plates 3, Figs. 60, \$7.50.

The third edition of Baly's *Spectroscopy* is to consist of four volumes, of which three have been published. The rapid development of the subject during the past two decades is shown by the fact that the first edition, a book of 550 pages published in 1905, was then considered a fairly adequate presentation on the practical side. The size of the book, on a subject at that time regarded as a department of optics, speaks for the very large amount of experimental work done during the second half of the last century. The development then attained in spectroscope design, especially the invention by Rowland of the concave grating in 1881, combined with the high ability of many of the investigators of that time, led to a rich accumulation of material, much of it of such quality that repetition has not been required.

The student of the present time, endeavoring to keep up in some degree with the flood of contributions since the "Bohr epoch" opened in 1913, is likely to have little time to acquaint himself at first hand with the earlier work, and may perhaps regard it as largely superseded. Professor Baly's ability to show how the older researches form the basis for the new does much to make the treatment attractive. The three editions, published in 1905, 1912, and 1924–1927, respectively, have given him an unusual opportunity to set forth the dependence of one period on another.

Professor Baly began his revision with the intention of making this third edition a work of two volumes. The Preface of Volume II, however, announces his larger plan for four volumes, of which the first two deal with general experimental work, while the third discusses the more direct applications of Bohr's theory to spectral series, the Zeeman effect, the Stark effect, and emission band spectra. The material promised for the fourth volume includes absorption spectra and displacements of spectral lines by pressure and by motion in the line of sight.

Volume I and the first chapter of Volume II are concerned with methods of observing spectra. While perhaps the specialist in any of the departments covered could point out desirable additional topics for discussion, it must be remembered that the treatise does not aim to be exhaustive. The material of the preceding edition is greatly amplified, and there is evidence everywhere of the extreme care Professor Baly has exercised in bringing the subject up to date, of his high talents for concise summary, and of his deep appreciation of the fascinating aspects of spectroscopy. His object has been to describe instruments of the latest type and methods of such general usefulness that they may be regarded as standard. The student will find here the basic theory of the instruments and a great store of information as to their use and the precautions to be observed. Especially valuable is the liberal inclusion of tables on the properties of optical materials, data useful in the computations of wave-lengths, the refractivity of air, ultimate lines, and standard wave-lengths, especially those of the iron spectrum. In the chapter on the infra-red and ultraviolet regions, the classic researches are well covered, with some description of the latest work. In this field, however, the student should keep closely in touch with the researches of the chief investigators, as methods of great usefulness are frequently being developed. The same is true of the first chapter in Volume II, on interference methods. Professor Baly justly lays stress on the great importance of interferometry in measurements of wave-length, and treats the theory in considerable detail. It may be noted, however, that Fabry and Buisson's method of using etalons with a grating spectrograph has been simplified by devices developed at the Bureau of Standards and at Mount Wilson, including the use of a concave grating in parallel light. The description in this chapter of the recent application of the Michelson interferometer to stellar investigations is of special interest.

It would have been well to omit the specifications of the "international iron arc" of 1913, which are given in two places, since the presence of pole-effect in this arc soon caused it to be replaced by the Pfund arc operated under specified conditions. The correct arrangement of the latter arc, when used for standards of wave-lengths, is given in Volume I, page 133, and Volume II, page 93.

Chapters ii and iii of Volume II, on methods of illumination and the nature of spectra, will put the student closely in touch with the field of experimental spectroscopy. There is perhaps at present a disproportionate number of workers on the theoretical side, while needed experimental data are very scanty in many fields, and in others the present results

need checking and extension. These two chapters are useful both as a working manual for the experimenter and to give those without first-hand experience some idea of the standard methods of producing spectra and the typical results which have been obtained.

The next chapter, on fluorescence and phosphorescence, including a treatment of resonance spectra, summarizes the data in this field, where there is much need of better support by theory.

A very valuable chapter on the photography of the spectrum concludes Volume II. Much of its material was contributed by Dr. Mees, of the Eastman Research Laboratory, and includes data on the absorption of various dyes useful in red and infra-red sensitometry and explicit directions for their use in sensitizing plates. This, with a discussion of ultra-violet photography, including mention of the use of fluorescent oil, gives material of the utmost value to those photographing the extreme regions of the spectrum.

In Volume III the value of the long chapter on series spectra must be considered with respect to the needs of the reader wishing to get hold of the present development of the subject, and also of the student who desires to gain working power in the analysis of spectral regularities. With allowance for the fact that it was written during a transition period when the life of an up-to-date book was limited to a few months, both classes of users will find the chapter of immense value. The scope of the treatment may be indicated by saying that it stops short of Schrödinger's wave-mechanics, that the brief introduction to the theory of Heisenberg and Hund should be supplemented by Hund's book and by recent applications of the theory, and that in the chapter of 307 pages Bohr's theory is introduced on page 63. We may be grateful that so large a fraction of the chapter is devoted to the older work. In many places it is pointed out that the new is the natural outgrowth of the old. Thus we are shown that Rydberg, working twenty-five years before Bohr, put forward three propositions which modern theory recognizes as having a sound basis. The work of Kayser and Runge had a definite place in the development, not only in furnishing high-grade wave-lengths, but in giving us a very full knowledge of the series systems characteristic of various types of lines for numerous elements. Baly traces and co-ordinates this work, and also the successive contributions of Ritz, Hicks, Paschen, and Fowler, so that we realize the extensive groundwork of spectroscopic knowledge that was laid for Bohr's theory and its various developments.

Beginning on page 63, we have a treatment of series spectra for the period since 1913, which in its arrangement and main features should give

the student a good introduction to modern methods. Bohr's application of his theory to hydrogen and helium is followed by the pioneer investigation of Fowler on series of enhanced lines. A discussion of Sommerfeld's quantization of plane elliptical orbits leads to a very clear explanation of quantum numbers, laying stress on the inner-quantum number, and such physical basis as we have for this quantity is brought out. Notation of series terms is always troublesome, and it would have been well to introduce as early as possible the subscripts based on the inner-quantum number. This change from Fowler's earlier notation is made on page 145.

The subject of multiplets in the complex spectra, the great advance in series classification during the past five years, is treated in much detail, with copious examples to enable the student to master the technique of the method. In this connection it might be well to state that in the analysis of complex spectra the term of lowest energy-level is taken as zero instead of beginning with the value of the series limit, and that many terms are found which have not been fitted into series, but can be detected by common separations, temperature class, intensities, or Zeeman effect. Valuable material on the intensities of lines in multiplets is supplied for the book by Professor Ornstein, who discusses the method developed by himself and his colleagues. Both the notation and the method of handling terms have been better systematized by the most recent work, and the papers by Russell on the titanium spectrum, published in this *Journal* during the last year, may be studied to advantage in this connection. Baly rounds out the chapter by a consideration of relativistic fine structure, X-ray spectra, and the work of Millikan and Bowen on stripped atoms.

The chapter on the Zeeman effect is really a continuation of the preceding. After a brief introduction, the chief features of the Landé theory are very clearly stated, and the application to spectral regularities is taken up in detail. It is impressive to those who remember the attempts to explain the complex Zeeman effect by means of the electron-magnetic theory, to read the statement on page 315 of what has been accomplished with the aid of the present theory.

In addition to the treatment of multiplet structure in connection with the Zeeman effect, an excellent description is given of the latest experimental methods, especially those of Back with the regular electro-magnet and a vacuum arc, and the use of very high momentary fields by Kapitza and Skinner. In view of the importance of research on the magnetic field, now that it has a firm theoretical basis, it is particularly advantageous to have this detailed presentation in English at this time.

The final chapters of Volume III, on the Stark effect and on emission band spectra, must be passed over briefly. The theoretical explanation of the Stark effect is well stated, and the need of more experimental data is made quite evident. The last chapter, in view of the great development of the theory of band spectra within the past two or three years, must be taken as an introduction to the comprehensive review by Birge and others, issued as *Bulletin of the National Research Council*, No. 57. Even so, enough is given to show the nature of the new era into which the investigation of band spectra has entered.

With the publication of the fourth volume, Professor Baly will have completed a work much needed at the present time; and all interested in spectroscopy, which number includes a very large proportion of physicists, chemists, and astronomers, may be grateful to him for taking the time in a busy life of research to make this very able and extended revision of a book which has been a standard reference work in its field.

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NOTICE TO CONTRIBUTORS

There is occasionally published in the *Astrophysical Journal* a Standing Notice (for instance, on pages 179–80 of the number for September, 1917). This is principally intended to guide contributors regarding the manuscript, illustrations, and reprints. This notice contains the following paragraph:

> Where unusual expense is involved in the publication of an article, on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution which he represents, according to a uniform system.

The present sheet has been printed for amplifying further that paragraph.

The "uniform system" according to which "arrangements are made" is as follows: The cost of composition in excess of $50, and of stock, presswork, and binding of pages in excess of 40 pages, for any one article shall be paid by the author or by the institution which he represents at the current rates of the University of Chicago Press. When four articles from one institution or author have appeared in any one volume, on which the cost of composition has amounted to $50 each, or when the total cost of composition incurred by the *Astrophysical Journal* on articles for one institution has reached the sum of $200, the entire cost of the composition, stock, presswork, and binding of any additional articles appearing in that volume shall be paid by the author or by the institution which he represents.

As to illustrations, the arrangement cannot be quite as specific, but it may be generally assumed that not more than three half-tone inserts can be allowed without payment by the author. The cost of paper, presswork, and binding for each full-page insert is about $8.00, aside from the cost of the half-tone itself. In the matter of zinc etchings, considerable latitude has to be allowed, as in many cases diagrams take the place of more expensive tables. It may be assumed, however, that it will seldom be possible for the *Journal* to bear an expense of over $25 for diagrams and text illustrations in any one article.

Contributors should notice that since January, 1917, it has been impossible to supply any free reprints of articles.

Reprints of articles, with or without covers, will be supplied to authors at cost. No reprints can be furnished unless a request for them is received before the *Journal* goes to press.

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. For details concerning the preparation of abstracts, see page 231 in the April, 1920, number of the *Journal*.

THE EDITORS